Decentralized Sliding Mode Voltage Control in DC Microgrids★

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Abstract— **The present paper deals with the design of a decentralized control scheme that relies on advanced control strategies of Sliding Mode (SM) type to regulate the voltage in islanded Direct Current (DC) microgrids. More specifically, the model of an islanded DC microgrid composed of several Distributed Generation units (DGus) interconnected according to an arbitrary topology including loops, is presented. The model takes into account the power lines dynamics and is affected by unknown load demand and unavoidable modelling uncertainties. First, a Second Order Sliding Mode (SOSM) control algorithm, belonging to the class of Suboptimal SOSM control, is proposed to solve the voltage control problem. Then, in order to obtain a continuous control signal that can be used as duty cycle of the power converter, a third order Sliding Mode (3-SM) control strategy is presented.**

I. Introduction

In the last decades economic, technological and environmental issues have encouraged the modification of the electricity generation and transmission towards smaller and Distributed Generation units (DGus). The increasing penetration of the Renewable Energy Sources (RES), such as photovoltaic panels or wind turbines, characterized by unpredictable generation, creates a new challenge for operating and controlling the power network safely and efficiently. This challenge can be addressed by exploiting the concept of "microgrids", which are clusters of DGus, loads and storage systems interconnected through power lines [1]. Moreover, they can operate disconnected from the main grid autonomously, in the so-called islanded operation mode [2].

In this context, due to the widespread use of Alternate Current (AC) electricity in most industrial, commercial and residential applications, the research mainly focused on AC microgrids (see e.g. [3]–[8] and the references there in). However, the development in power electronics technology, which enables DC voltage transformation into different levels, and the increasing number of DC loads and applications (e.g. electronic appliances and electric vehicles) in several fields (e.g. automotive, marine, avionics [9]), are moving the interest towards DC microgrids. Indeed, several factors favour the use of DC-based power systems. Foremost, DC distribution systems are more efficient than AC distribution [10].

In the literature, the voltage control problem in DC microgrids has been treated and solved with different control approaches. In [11] a droop controller is proposed to regulate the microgrid voltage in order to achieve load sharing. A fuzzy control strategy is designed in [12], while [13] proposes fuzzy methodology with gain-scheduling techniques to accomplish both power sharing and energy management. Instead in [14] a model predictive control-based Maximum Power Point Tracking and droop current controller are designed in order to interface photovoltaic panels in smart DC distribution systems.

In this paper an islanded DC microgrid with DGus interconnected according to an arbitrary complex and meshed topology including loops is considered, and each DGu is interfaced with the network through a DC-DC Buck converter. The power network is represented by a connected and undirected graph, and the model, that takes into account the power lines dynamics, is affected by unknown load demand and unavoidable modelling uncertainties.

In order to solve the aforementioned voltage control problem, Sliding Mode (SM) control methodology is applied. SM control belongs to the class of Variable Structure Control Systems so that it seems perfectly adequate to control the variable structure nature of DC-DC converters [15]. Moreover, SM control is very appreciated for its robustness properties against a wide class of unavoidable modelling uncertainties and external disturbances [16], [17]. In particular, we first propose a second order sliding mode control algorithm belonging to the class of Suboptimal SOSM (SSOSM) control [18]. However, this solution allows the switching frequency of the Buck converter to be not constant and not a priori fixed. So, the switching frequency could be very high, implying the increase of the power losses. Then, in order to avoid this problem and obtain a continuous control signal that can be used as duty cycle of the Buck converter switch, a third order Sliding Mode (3-SM) control [19] is proposed. Both the proposed solutions are very easy to implement, since the sliding variable uses only the measurement of the load voltage locally, so that the control approach is decentralized. Finally, the proposed solutions are theoretically analyzed and assessed in simulation, proving the asymptotic stability of the whole microgrid.

The present paper is organized as follows. In Section II the microgrid model is presented and the control problem is formulated, while in Section III the control strategies are proposed. In Section IV the stability properties of the controlled system are analyzed, while in Section V the simu-

★ This is the final version of the accepted paper submitted for inclusion in the Proceedings of the American Control Conference, Seattle, WA, USA, May 2017.

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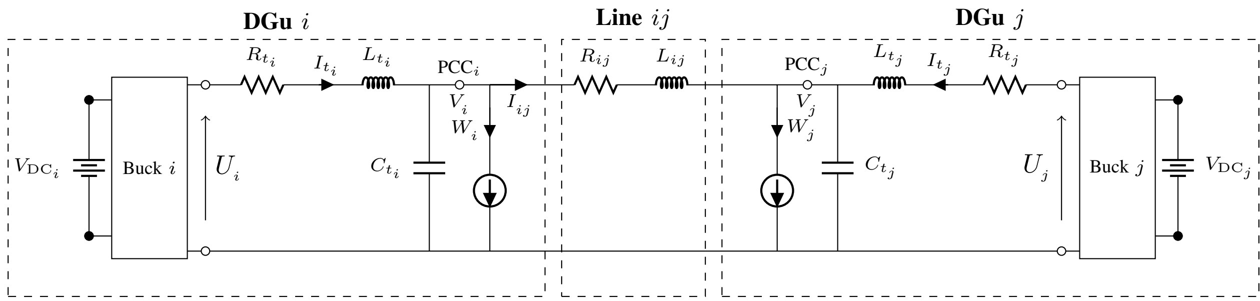

Fig. 1. The considered electrical diagram of a typical DC microgrid composed of two DGUs.

lation results are illustrated and discussed. Some conclusions are finally gathered in Section VI.

II. Problem Formulation

Consider the schematic electrical diagram of a typical microgrid composed of two DGus in Fig. 1. The renewable energy source of a DGu is represented by a DC voltage source V_{DC}, and it is interfaced with the electric DC network through a DC-DC Buck converter that supplies a DC local load. The local DC load is connected to the so-called Point of Common Coupling (PCC) and it can be treated as a current disturbance W. At the output of the Buck converter a low-pass filter $R_tL_tC_t$ is considered, where R_t represents the filter parasitic resistance. Moreover, the DGu$_i$ can exchange power with the DGu$_j$ through the resistive-inductive line $R_{ij}L_{ij}$.

The network is represented by a connected and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the nodes $\mathcal{V} = \{1, ..., n\}$, represent the DGus and the edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V} = \{1, ..., m\}$, represent the distribution lines interconnecting the DGus. First, consider the scheme reported in Figure 1. By applying the Kirchhoff's current (KCL) and voltage (KVL) laws, the governing dynamic equations[1] of the i-th node are the following

$$\begin{aligned} \tfrac{d}{dt} I_{t_i} &= -\tfrac{R_{t_i}}{L_{t_i}} I_{t_i} - \tfrac{1}{L_{t_i}} V_i + \tfrac{1}{L_{t_i}} U_i \\ \tfrac{d}{dt} V_i &= \tfrac{1}{C_{t_i}} I_{t_i} - \tfrac{1}{C_{t_i}} W_i - \tfrac{1}{C_{t_i}} \sum_{j \in \mathcal{N}_i} I_{ij}, \end{aligned} \tag{1}$$

where $\mathcal{N}_i$ is the set of nodes (i.e., DGus) connected to the i-th node by distribution lines. Moreover, for each $j \in \mathcal{N}_i$, the line dynamics can be expressed as

$$\tfrac{d}{dt} I_{ij} = \tfrac{1}{L_{ij}} (V_i - V_j) - \tfrac{R_{ij}}{L_{ij}} I_{ij} \ . \tag{2}$$

Now, the network topology can be represented by its corresponding incidence matrix $D \in \mathbb{R}^{n \times m}$. The ends of edge k are arbitrary labeled with a '+' and a '-'. More precisely, one has that

$$D_{ik} = \begin{cases} +1 & \text{if } I_k \text{ entering into DGu}_i \text{ is assumed positive} \\ -1 & \text{if } I_k \text{ exiting from DGu}_i \text{ is assumed positive} \\ 0 & \text{if } k \text{ is not connected to } i, \end{cases}$$

[1]For the sake of simplicity, the dependence of all the variables on time t is omitted throughout the paper.

$I_k = I_{ij}$ being the current exchanged through the edge k (i.e., the distribution line $R_{ij}L_{ij}$) of the graph $\mathcal{G}$.

To study the overall microgrid we write system (1) and the distribution lines dynamics in (2) compactly for all nodes $i \in \mathcal{V}$ as

$$\begin{aligned} \tfrac{d}{dt} I_t &= -L_t^{-1} R_t I_t - L_t^{-1} V + L_t^{-1} U \\ \tfrac{d}{dt} V &= C_t^{-1} I_t + C_t^{-1} D I - C_t^{-1} W \\ \tfrac{d}{dt} I &= -L^{-1} D^T V - L^{-1} R I, \end{aligned} \tag{3}$$

where $V \in \mathbb{R}^n$, $I_t \in \mathbb{R}^n$, $W \in \mathbb{R}^n$, $I \in \mathbb{R}^m$, and $U \in \mathbb{R}^n$ represent, respectively, the following signals: the load voltages, the currents generated by the DGus, the unknown currents demanded by the loads, the currents along the interconnecting lines, and the Buck converters output voltages. Moreover C_t, L_t and R_t are $n \times n$ diagonal matrices, while L and R are $m \times m$ diagonal matrices, e.g. $R_t = \mathrm{diag}\{R_{t_1}, \ldots, R_{t_n}\}$ and $R = \mathrm{diag}\{R_1, \ldots, R_m\}$, with $R_k = R_{ij}$. In the following we use $x_{[S]}$ to denote the vector $[S_1, \ldots, S_n]^T$ with $S \in \{V, I_t\}$, and $x_{[I]}$ to denote the vector $[I_1, \ldots, I_m]^T$, with $I_k = I_{ij}$.

Now, system (3) can be written in the state-space representation, i.e.,

$$\begin{aligned} \dot{x}_{[I_t]} &= -L_t^{-1} R_t x_{[I_t]} - L_t^{-1} x_{[V]} + L_t^{-1} u \\ \dot{x}_{[V]} &= C_t^{-1} x_{[I_t]} + C_t^{-1} D x_{[I]} - C_t^{-1} w \\ \dot{x}_{[I]} &= -L^{-1} D^T x_{[V]} - L^{-1} R x_{[I]} \\ y &= x_{[V]}, \end{aligned} \tag{4}$$

where $x = \left[x_{[I_t]}^T \, x_{[V]}^T \, x_{[I]}^T \right]^T \in \mathbb{R}^{2n+m}$ is the state variables vector, $u = U \in \mathbb{R}^n$ is the control variables vector, $w = W \in \mathbb{R}^n$ is the disturbances vector, and $y = x_{[V]} \in \mathbb{R}^n$ is the controlled variables vector. Then, the previous system can be written as

$$\begin{aligned} \dot{x} &= Ax + Bu + B_w w \\ y &= Cx, \end{aligned} \tag{5}$$

where $A \in \mathbb{R}^{(2n+m) \times (2n+m)}$ is the dynamics matrix of the microgrid, $B \in \mathbb{R}^{(2n+m) \times n}$, and $B_w \in \mathbb{R}^{(2n+m) \times n}$, and $C \in \mathbb{R}^{n \times (2n+m)}$. To permit the controller design, the following assumption is required on the state and the disturbance.

Assumption 1 *The load voltage V_i is locally available at DGu$_i$. The disturbance W_i is unknown but bounded, of class* C^2*, with bounded first and second time derivatives.*

Now we are in a position to formulate the control problem: *Let Assumption 1 hold. Given system* (1)-(5)*, design a decentralized control scheme capable of guaranteeing that the tracking error between any controlled variable and the corresponding reference is steered to zero in a finite time in spite of the uncertainties, such that the overall system is asymptotically stable.*

III. The Proposed Sliding Mode Control Schemes

In this section, the SSOSM and 3-SM control are proposed to solve the aforementioned voltage control problem.

A. Suboptimal SOSM Controller

Consider the state-space model (5) and select the sliding variables vector as

$$\sigma = y - y^\star, \tag{6}$$

where $\sigma \in \mathbb{R}^n$, and $y^\star = x^\star_{[V]} \in \mathbb{R}^n$ is the vector of reference values, such that the following assumption is verified.

Assumption 2 *Let the references $y_i^\star$, $i = 1, \ldots, n$, be of class* C^3*, with first and second time derivatives Lipschitz continuous.*

Moreover, with reference to (6), it appears that the relative degree[2] is equal to 2, so that a SOSM control naturally applies [18], [20]. According to the SOSM control theory, the auxiliary variables vectors $\xi_1 = \sigma$ and $\xi_2 = \dot{\sigma}$ have to be defined and the corresponding auxiliary system can be expressed as

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= f + gu, \end{aligned} \tag{7}$$

where ξ_2 is not measurable since, according to Assumption 1, w is unknown. More specifically, one has that

$$\begin{aligned} f \;=\;& -C_t^{-1}(L_t^{-1} + DL^{-1}D^T)x_{[V]} \\ & -C_t^{-1}L_t^{-1}R_t x_{[I_t]} - C_t^{-1}DL^{-1}Rx_{[I]} \\ & -C_t^{-1}\dot{w} - \ddot{x}^\star_{[V]} \\ g \;=\;& C_t^{-1}L_t^{-1}, \end{aligned} \tag{8}$$

where $f \in \mathbb{R}^n$, $g \in \mathbb{R}^{n\times n}$ are uncertain but bounded, i.e.,

$$|f_i| \le F_i, \quad G_{\min_i} \le g_{ii} \le G_{\max_i}, \quad i = 1, \ldots, n, \tag{9}$$

F_i, $G_{\min_i}$ and $G_{\max_i}$ being known positive constants. The i-th control law, which is used to steer ξ_{1_i} and ξ_{2_i}, $i = 1, \ldots, n$, to zero in a finite time in spite of the uncertainties, in analogy with [18], can be expressed as follows

$$u_i = -\alpha_i U_{\max_i} \operatorname{sgn}\left(\xi_{1_i} - \tfrac{1}{2}\xi_{1_{\max_i}}\right), \tag{10}$$

with bounds

$$U_{\max_i} > \max\left(\frac{F_i}{\alpha_i^* G_{\min_i}}; \frac{4F_i}{3G_{\min_i} - \alpha_i^* G_{\max_i}}\right) \tag{11}$$

$$\alpha_i^* \in (0,1] \cap \left(0, \frac{3G_{\min_i}}{G_{\max_i}}\right). \tag{12}$$

Remark 1 *Note that, in real applications, the discontinuous control can be directly used to open and close the switch of the Buck converter. More precisely, when in* (10) $\operatorname{sgn}\left(\xi_{1_i} - \frac{1}{2}\xi_{1_{\max_i}}\right) = -1$*, the switch is closed and the Buck output voltage is V_{DC_i}. Otherwise, when* $\operatorname{sgn}\left(\xi_{1_i} - \frac{1}{2}\xi_{1_{\max_i}}\right) = 1$*, the switch is open and the Buck output voltage is zero. However, the IGBTs (Insulated Gate Bipolar Transistor) switching frequency is not constant and cannot be fixed. Indeed, the switching frequency could be very high, implying the increase of the power losses.*

B. 3-SM Controller

Usually, to control Buck converters, the Pulse Width Modulation (PWM) technique with constant switching frequency is used. To do this, a continuous control signal that represents the duty cycle of the switch of the Buck converter is required. In order to obtain a continuous control signal, as suggested in [20], the system relative degree can be artificially increased. Therefore, by defining the auxiliary variables vectors $\xi_1 = \sigma$, $\xi_2 = \dot{\sigma}$ and $\xi_3 = \ddot{\sigma}$, the auxiliary system can be expressed as

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \xi_3 \\ \dot{\xi}_3 &= \varphi + \gamma h \\ \dot{u} &= h, \end{aligned} \tag{13}$$

where ξ_2 and ξ_3 are unmeasurable and

$$\begin{aligned} \varphi =\;& +C_t^{-1}(L_t^{-2}R_t + DL^{-2}RD^T)x_{[V]} \\ & +C_t^{-1}(L_t^{-2}R_t^2 - (L_t^{-1} + DL^{-1}D^T)C_t^{-1})x_{[I_t]} \\ & +C_t^{-1}(DL^{-2}R^2 - (L_t^{-1} + DL^{-1}D^T)C_t^{-1}D)x_{[I]} \\ & -C_t^{-1}L_t^{-2}R_t u + C_t^{-1}(L_t^{-1} + DL^{-1}D^T)C_t^{-1}w \\ & -C_t^{-1}\ddot{w} - x^{\star(3)}_{[V]} \\ \gamma =\;& C_t^{-1}L_t^{-1} \end{aligned} \tag{14}$$

are uncertain with bounds

$$|\varphi_i| \le \Phi_i, \quad \Gamma_{\min_i} \le \gamma_{ii} \le \Gamma_{\max_i}, \quad i = 1, \ldots, n, \tag{15}$$

Φ_i, $\Gamma_{\min_i}$ and $\Gamma_{\max_i}$ being known positive constants.

Now, the third order Sliding Mode (3-SM) control law proposed in [19] can be used to steer ξ_{1_i}, ξ_{2_i} and ξ_{3_i}, $i =$

[2]The relative degree is the minimum order r of the time derivative $\sigma_i^{(r)}, i = 1, \ldots, n$, of the sliding variable associated to the i-th node in which the control $u_i, i = 1, \ldots, n$, explicitly appears.

$1, \dots, n$, to zero in a finite time in spite of the uncertainties, i.e.,

$$h_i = -\alpha_i \begin{cases} h_{1_i} = \mathrm{sgn}(\ddot{\sigma}_i) & \bar{\sigma}_i \in M_{1_i}/M_{0_i} \\ h_{2_i} = \mathrm{sgn}\left(\dot{\sigma}_i + \frac{\ddot{\sigma}_i^2 h_{1_i}}{2\alpha_{r_i}}\right) & \bar{\sigma}_i \in M_{2_i}/M_{1_i} \\ h_{3_i} = \mathrm{sgn}(s_i(\bar{\sigma}_i)) & \text{else,} \end{cases} \tag{16}$$

with $\bar{\sigma}_i = [\sigma_i, \dot{\sigma}_i, \ddot{\sigma}_i]^T$ and

$$s_i(\bar{\sigma}_i) = \sigma_i + \frac{\ddot{\sigma}_i^3}{3\alpha_{r_i}^2} + h_{2_i}\left[\frac{1}{\sqrt{\alpha_{r_i}}}\left(h_{2_i}\dot{\sigma}_i + \frac{\ddot{\sigma}_i^2}{2\alpha_{r_i}}\right)^{\frac{3}{2}} + \frac{\dot{\sigma}_i\ddot{\sigma}_i}{\alpha_{r_i}}\right],$$

with

$$\alpha_{r_i} = \alpha_i \Gamma_{\min_i} - \Phi_i > 0. \tag{17}$$

In (16) the manifolds M_{1_i}, M_{2_i}, M_{3_i} are defined as

$$\begin{array}{rcl} M_{0_i} & = & \left\{\bar{\sigma}_i \in \mathbb{R}^3 : \sigma_i = \dot{\sigma}_i = \ddot{\sigma}_i = 0\right\} \\ M_{1_i} & = & \left\{\bar{\sigma}_i \in \mathbb{R}^3 : \sigma_i - \frac{\ddot{\sigma}_i^3}{6\alpha_{r_i}^2} = 0, \dot{\sigma}_i + \frac{\ddot{\sigma}_i|\ddot{\sigma}_i|}{2\alpha_{r_i}} = 0\right\} \\ M_{2_i} & = & \left\{\bar{\sigma}_i \in \mathbb{R}^3 : s_i(\bar{\sigma}_i) = 0\right\}. \end{array} \tag{18}$$

Note that the control signal $h_i = \dot{u}_i$ is discontinuous and affects only $\sigma_i^{(3)}$, while the control actually fed into the plant u_i is continuous. Then, in real applications it can be used as duty cycle of the switch of the i-th Buck converter.

From (16), one can also observe that the controller of DGu$_i$ requires not only σ_i, but also $\dot{\sigma}_i$ and $\ddot{\sigma}_i$. Yet, according to Assumption 1, only the load voltage V_i is measurable at DGu$_i$. Then, one can rely on Levant's second-order differentiator [21] to retrieve $\dot{\sigma}_i$ and $\ddot{\sigma}_i$ in a finite time. With reference to system (13), for $i = 1, \dots, n$, one has

$$\begin{array}{rcl} \dot{\hat{\xi}}_{1_i} & = & -\lambda_{0_i}\left|\hat{\xi}_{1_i} - \xi_{1_i}\right|^{\frac{2}{3}} \mathrm{sgn}\left(\hat{\xi}_{1_i} - \xi_{1_i}\right) + \hat{\xi}_{2_i} \\ \dot{\hat{\xi}}_{2_i} & = & -\lambda_{1_i}\left|\hat{\xi}_{2_i} - \dot{\hat{\xi}}_{1_i}\right|^{\frac{1}{2}} \mathrm{sgn}\left(\hat{\xi}_{2_i} - \dot{\hat{\xi}}_{1_i}\right) + \hat{\xi}_{3_i} \\ \dot{\hat{\xi}}_{3_i} & = & -\lambda_{2_i}\,\mathrm{sgn}\left(\hat{\xi}_{3_i} - \dot{\hat{\xi}}_{2_i}\right), \end{array} \tag{19}$$

where $\hat{\xi}_{1_i}$, $\hat{\xi}_{2_i}$, $\hat{\xi}_{3_i}$ are the estimated values of $\xi_{1_i}, \xi_{2_i}, \xi_{3_i}$, respectively, and $\lambda_{0_i} = 3\Lambda_i^{1/3}$, $\lambda_{1_i} = 1.5\Lambda_i^{1/2}$, $\lambda_{2_i} = 1.1\Lambda_i$, $\Lambda_i > 0$, is a possible choice of the differentiator parameters suggested in [21].

IV. Stability Analysis

With reference to the proposed Higher Order SM (HOSM) control approaches, the following results can be proved. For the sake of brevity the corresponding proofs are omitted.

Lemma 1 *Given the auxiliary system* (7)*, by applying the SSOSM Algorithm* (10)-(12)*, the sliding variables σ_i and their first time derivatives $\dot{\sigma}_i$, $i = 1, \dots, n$, are steered to zero in a finite time, in spite of the uncertainties.*

Let $\tilde{x}$ be the error given by the difference between the state and the equilibrium point associated to the reference $y^\star$ when w is constant. Therefore, $\forall\, t \geq t_{\mathrm{r}}$, the error system can be expressed as

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}_{\mathrm{eq}}. \tag{20}$$

Now, relying on Lemma 1, one can prove the major result concerning the evolution of the considered DC microgrid controlled via the proposed decentralized control scheme.

Theorem 1 *Consider system* (1)-(5) *and the sliding variable* (6) *controlled via the SSOSM Algorithm* (10)-(12)*. Given a constant reference $y^\star$ and a constant disturbance w, $\forall\, t \geq t_{\mathrm{r}}$, $\forall\, x(t_{\mathrm{r}}) \in \mathbb{R}^{2n+m}$, the origin of the error system* (20) *is a robust asymptotically stable equilibrium point.*

Now, with reference to the proposed 3-SM control law, the following results can be proved.

Lemma 2 *Given the auxiliary system* (13)*, assume $t_0 \geq t_{\mathrm{Ld}}$, t_0, t_{Ld} being the initial time instant and the finite time necessary for the Levant's differentiator convergence, respectively. By applying the 3-SM control law* (16)-(18)*, the sliding variables σ_i and their first and second time derivatives $\dot{\sigma}_i, \ddot{\sigma}_i$, $i = 1, \dots, n$, are steered to zero in a finite time $t_{\mathrm{r}} \geq t_0$, in spite of the uncertainties.*

Theorem 2 *Consider system* (1)-(5) *and the sliding variable* (6) *controlled via the 3-SM control law* (16)-(18)*. Given a constant reference $y^\star$ and a constant disturbance w, $\forall\, t \geq t_{\mathrm{r}} \geq t_0 \geq t_{\mathrm{Ld}}$, $\forall\, x(t_{\mathrm{r}}) \in \mathbb{R}^{2n+m}$, the origin of the error system* (20) *is a robust asymptotically stable equilibrium point.*

V. Simulation Results

In this section, the proposed decentralized control scheme is assessed in simulation by implementing the realistic model of a DC islanded microgrid composed of 5 DGus (n =5) with meshed topology (m =7), as depicted in Figure 2. The

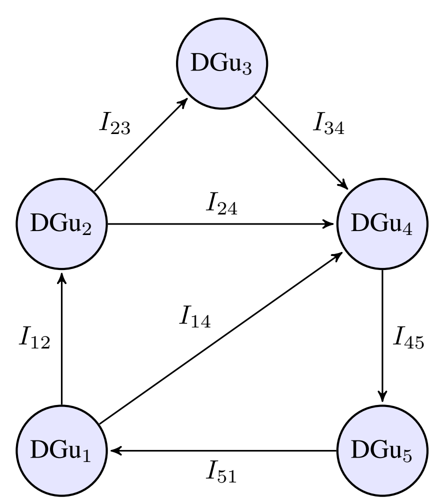

Fig. 2. Scheme of the considered DC microgrid composed of 5 DGus. The arrows indicate the positive direction of the currents through the power network.

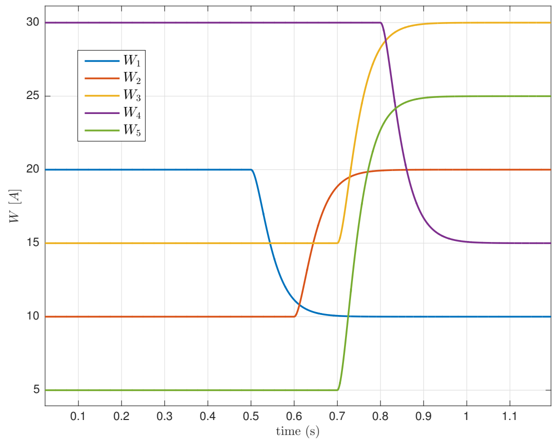

Fig. 3. Load currents treated as disturbances.

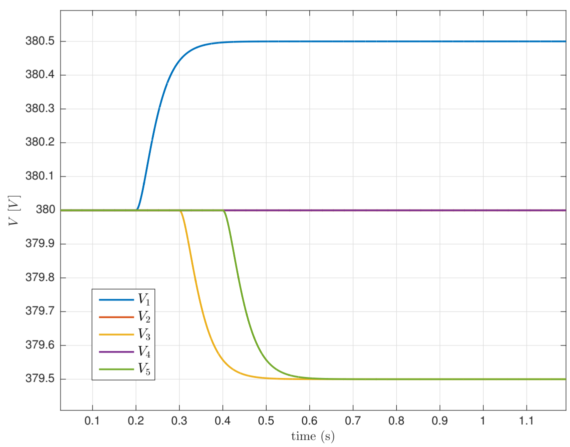

Fig. 4. Load voltages in the presence of disturbance and reference variations.

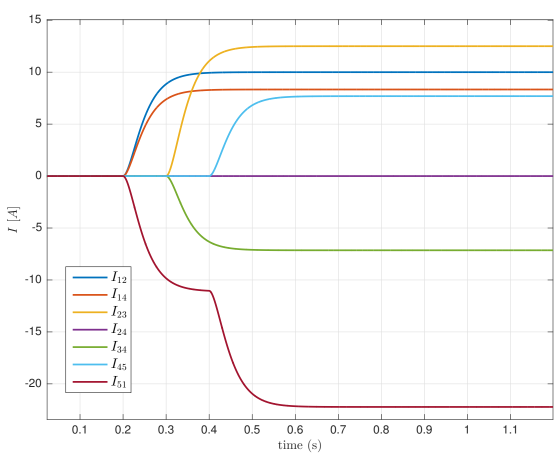

Fig. 5. The currents through the distribution lines.

TABLE I

BUCK FILTER PARAMETERS

	R_t [Ω]	L_t [mH]	C_t [mF]
DGu_1	0.2	1.8	2
DGu_2	0.1	1.6	2.1
DGu_3	0.3	2	1.8
DGu_4	0.4	2.1	1.9
DGu_5	0.5	1.9	2.2

TABLE II

LINE PARAMETERS

	R [mΩ]	L [µH]
$Line_{12}$	50	1.9
$Line_{14}$	60	2
$Line_{23}$	40	1.7
$Line_{24}$	80	2.1
$Line_{34}$	70	1.8
$Line_{45}$	65	1.6
$Line_{51}$	45	2

TABLE III

INITIAL VALUES AND VARIATIONS OF LOADS AND REFERENCES

	W_i [A]	t [s]	ΔW_i [A]	$V_i^\star$ [V]	t [s]	$\Delta V_i^\star$ [V]
DGu_1	20	0.5	-10	380	0.2	+0.5
DGu_2	10	0.6	+10	380	-	-
DGu_3	15	0.7	+15	380	0.3	-0.5
DGu_4	30	0.8	-15	380	-	-
DGu_5	5	0.7	+20	380	0.4	-0.5

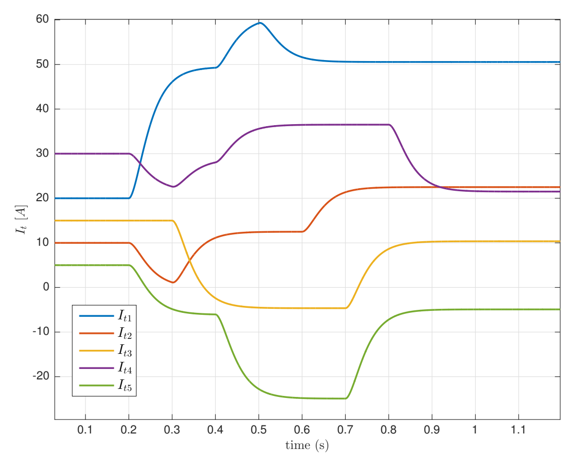

Fig. 6. Generated currents (at the output of the L_tC_t filter).

incidence matrix $D \in \mathbb{R}^{5\times 7}$, which describes the network topology, can be expressed as

$$D = \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

The electrical parameters of the output filters of the Buck converters are reported in Table I, while the parameters of the interconnecting distribution lines are reported in Table II. The performances of the proposed decentralized control scheme are validated considering unknown load dynamics and voltage reference changes. All the variations are reported in Table III and they are such that Assumptions 1 and 2 hold.

For the sake of brevity, we show only the simulation results obtained by applying the 3-SM control law. Similar results can be obtained by using the SSOSM control algorithm, yet, in that case, a higher switching frequency is required. In Figure 3 the time evolution of the load currents are reported, while in Figure 4 the load voltages are shown. In Figure 4 one can observe the robustness of the proposed decentralized control approach with respect to both reference and load variations. Moreover, the voltage dynamics of each DGu is not affected neither by load nor by reference variations in the neighbouring DGus. In Figure 5 the time evolution of the currents flowing through the distribution lines interconnecting the DGus is illustrated. In particular, one can observe that there is current exchange when two adjacent DGus have different PCC voltage values. Finally Figure 6 shows the time evolution of the currents generated by each DGu (at the output of the L_tC_t filter).

VI. Conclusions and Future Research

In this paper a decentralized control scheme based on Higher Order Sliding Mode control strategies is designed to regulate the voltage in islanded DC microgrids with arbitrary complex topology. A DC microgrid composed of several interconnected Distributed Generation Units, power lines and loads is modelled, and the power network is represented by a connected and undirected graph. In particular, SOSM and 3-SM control strategies are used to stabilize the microgrid voltage in spite of unavoidable modelling uncertainties and unknown load dynamics. The chattering alleviation performed by the 3-SM control algorithm allows one to obtain a continuous control signal that can be used in PWM technique as duty cycle of the switch of the Buck converter in order to attain a constant switching frequency. The asymptotic stability of the whole system is proved, and the performance of the proposed decentralized control approach is evaluated in simulation considering a DC microgrid composed of five DGus arranged in a meshed topology including loops.

An interesting extension to the presented work could be the design of a distributed networked control scheme in order to obtain power sharing among the DGus, even by using event-triggered sliding mode methodology [22].

References

[1] R. Lasseter and P. Paigi, "Microgrid: a conceptual solution," in *Proc. 35th IEEE Power Electronics Specialists Conf.*, vol. 6, Aachen, Germany, June 2004, pp. 4285–4290.

[2] F. Katiraei, M. Iravani, and P. Lehn, "Micro-grid autonomous operation during and subsequent to islanding process," *IEEE Trans. Power Del.*, vol. 20, no. 1, pp. 248–257, Jan. 2005.

[3] M. S. Sadabadi, Q. Shafiee, and A. Karimi, "Plug-and-play voltage stabilization in inverter-interfaced microgrids via a robust control strategy," *IEEE Trans. Control Syst. Techn.*, vol. PP, no. 99, pp. 1–11, July 2016.

[4] M. Cucuzzella, G. P. Incremona, and A. Ferrara, "Master-slave second order sliding mode control for microgrids," in *Proc. American Control Conf.*, Chicago, IL, USA, July 2015, pp. 5188–5193.

[5] ——, "Third order sliding mode voltage control in microgrids," in *Proc. European Control Conf.*, Linz, Austria, July 2015, pp. 2384 – 2389.

[6] ——, "Design of robust higher order sliding mode control for microgrids," *IEEE J. Emerg. Sel. Topics Circuits Syst.*, vol. 5, no. 3, pp. 393–401, Sept. 2015.

[7] G. P. Incremona, M. Cucuzzella, and A. Ferrara, "Adaptive suboptimal second-order sliding mode control for microgrids," *Int. J. Control*, pp. 1–19, Jan. 2016.

[8] M. Cucuzzella, G. P. Incremona, M. Guastalli, and A. Ferrara, "Sliding mode control for maximum power point tracking of photovoltaic inverters in microgrids," in *Proc. IEEE 55th Conf. Decision Control*, Dec. 2016, pp. 7294–7299.

[9] A. Cavallo, B. Guida, A. Buonanno, and E. Sparaco, "Smart buck-boost converter unit operations for aeronautical applications," in *Proc. IEEE 54th Conf. Decision Control*, Osaka, Japan, Dec. 2015, pp. 4734–4739.

[10] J. J. Justo, F. Mwasilu, J. Lee, and J.-W. Jung, "Ac-microgrids versus dc-microgrids with distributed energy resources: A review," *Renewable Sustain. Energy Rev.*, vol. 24, pp. 387–405, Aug. 2013.

[11] R. A. F. Ferreira, H. A. C. Braga, A. A. Ferreira, and P. G. Barbosa, "Analysis of voltage droop control method for dc microgrids with simulink: Modelling and simulation," in *Proc. IEEE/IAS 10th Industry Applications Int. Conf.*, Nov. 2012, pp. 1–6.

[12] Y. K. Chen, Y. C. Wu, C. C. Song, and Y. S. Chen, "Design and implementation of energy management system with fuzzy control for dc microgrid systems," *IEEE Trans. Power Electronics*, vol. 28, no. 4, pp. 1563–1570, Apr. 2013.

[13] H. Kakigano, Y. Miura, and T. Ise, "Distribution voltage control for dc microgrids using fuzzy control and gain-scheduling technique," *IEEE Trans. Power Electronics*, vol. 28, no. 5, pp. 2246–2258, May 2013.

[14] M. B. Shadmand, R. S. Balog, and H. Abu-Rub, "Model predictive control of pv sources in a smart dc distribution system: Maximum power point tracking and droop control," *IEEE Trans. Energy Convers.*, vol. 29, no. 4, pp. 913–921, Dec. 2014.

[15] S.-C. Tan, Y. M. Lai, M. K. H. Cheung, and C. K. Tse, "On the practical design of a sliding mode voltage controlled buck converter," *IEEE Trans. Power Electronics*, vol. 20, no. 2, pp. 425–437, Mar. 2005.

[16] V. I. Utkin, *Sliding Modes in Optimization and Control Problems*. New York: Springer Verlag, 1992.

[17] C. Edwards and S. K. Spurgen, *Sliding Mode Control: Theory and Applications*. London, UK: Taylor and Francis, 1998.

[18] G. Bartolini, A. Ferrara, and E. Usai, "Chattering avoidance by second-order sliding mode control," *IEEE Trans. Autom. Control*, vol. 43, no. 2, pp. 241–246, Feb. 1998.

[19] F. Dinuzzo and A. Ferrara, "Higher order sliding mode controllers with optimal reaching," *IEEE Trans. Autom. Control*, vol. 54, no. 9, pp. 2126 –2136, Sept. 2009.

[20] G. Bartolini, A. Ferrara, and E. Usai, "Output tracking control of uncertain nonlinear second-order systems," *Automatica*, vol. 33, no. 12, pp. 2203 – 2212, Dec. 1997.

[21] A. Levant, "Higher-order sliding modes, differentiation and output-feedback control," *Int. J. Control*, vol. 76, no. 9-10, pp. 924–941, Jan. 2003.

[22] M. Cucuzzella and A. Ferrara, "Event-triggered second order sliding mode control of nonlinear uncertain systems," in *Proc. European Control Conf.*, Aalborg, Denmark, June 2016, pp. 295–300.