Distributed control of DC grids: a social perspective

M. Cucuzzella, K. C. Kosaraju, T. Bouman, G. Schuitema, S. Johnson-Zawadzki, C. Fischione, L. Steg, J. M. A. Scherpen

***Abstract*—In this paper, a novel distributed control strategy addressing a (feasible) social-physical welfare problem in Direct Current (DC) smart grids is proposed, which is based on physical, technical and social aspects of the grid. Firstly, we formulate a (convex) optimization problem that allows prosumers to share power – and the financial and psycho-social costs and benefits associated with the generation and consumption of power – with each other, taking into account the technical, physical and social aspects and constraints of the grid (e.g., stability, safety, user preferences). Secondly, we design a controller whose (unforced) dynamics represent the continuous time primal-dual dynamics of the considered optimization problem. Thirdly, a passive interconnection between the physical grid and the controller is presented. Global asymptotic convergence of the closed-loop system to the desired steady-state is proved and simulations illustrate and confirm the theoretical results.**

***Index Terms*—Distributed control, DC power systems, Social factors.**

I. Introduction

TRANSITIONING towards 100% renewable energy systems brings about many challenges, requiring solutions that consider physical, technical as well as social aspects of energy systems . Even if renewable energy technologies are able to deliver the energy demanded, changes in end-users energy-related choices, roles and behaviors are needed to guarantee the efficiency and sustainability of the energy system. For instance, the generation (e.g., photovoltaic technology, wind turbines) and storage (e.g., home batteries, electric vehicles) of power at household level is far more efficient and sustainable if prosumers share and cooperate within the (local) energy system, limiting the excess of generated energy, preventing risks associated with single suppliers, and reducing financial and environmental costs related to the production and installation of these technologies [1].

In the current paper, we focus on the optimization of current-sharing, the situation in which prosumers fairly share their generated current or stored energy from renewable sources with peers within their local electricity network. Importantly, given the central role of end-users in renewable energy systems, we not only look at the commonly considered technical and physical aspects of energy systems when designing the control scheme, but also pioneer with integrating social aspects in our energy models. Through this approach, we provide first insights in, and promote and facilitate, the so needed integration of more social aspects in the modeling and optimization of energy systems (e.g., [2], [3]).

From a technical perspective, the recent wide spread of renewable energy sources motivates the design and operation of Direct Current (DC) smart grids [4], [5], [6], [7], [8], which are interconnected clusters of prosumers interacting with each other through distributed transmission lines . To guarantee a proper and safe functioning of the power network, voltage stabilization is the main goal to achieve in DC smart grids [9]. Additionally, to avoid the overstressing of a single energy source, it is generally desired that the total demand is shared in a fair way among all the prosumers of the smart grid [1]. However, to permit prosumers to share their generated current or power, voltage differences among the nodes of the smart grid are necessary. As a consequence, it is generally not possible to achieve the aforementioned objectives simultaneously.

In the literature, several control techniques have been proposed to control the voltages towards the corresponding nominal values (see for instance [10] and the references therein). Other works have proposed consensus-based control schemes achieving current/power sharing without regulating the voltage (see for instance [11] and the references therein). Differently from the above mentioned works, consensus-based protocols have been recently designed for achieving both current sharing and a peculiar form of voltage regulation, where the average value of the voltages of the whole microgrid is controlled towards a desired setpoint (see for instance [1], [12] and the references therein). However, regulating *only* the average voltage may introduce, in some nodes of the microgrid, large voltage deviations from the corresponding nominal value, making this solution not always adequate in practical applications. This motivated us to design an optimal control scheme aiming to share, among the prosumers of the smart grid, the largest possible amount of the (controllable) total demand in compliance with physical constraints that ensure safety and reliability (a similar control problem is addressed in [13] by synthesizing a centralized symbolic controller), while considering social aspects of the involved prosumers as well.

Critically, the extent to which prosumers accept such solutions, and are thus willing to share power and (controllable) demand, strongly depends on these prosumers' personal preferences and motives. Specifically, research in social sciences identified four key values (i.e., general life goals that motivate and steer individuals' behaviours) that are particularly relevant

M. Cucuzzella, K. C. Kosaraju and J. M. A. Scherpen are with the Jan C. Wilems Center for Systems and Control, ENTEG, Faculty of Science and Engineering, University of Groningen, Nijenborgh 4, 9747 AG Groningen, the Netherlands (email: {m.cucuzzella, k.c.kosaraju, j.m.a.scherpen}@rug.nl).

T. Bouman, S. Johnson-Zawadzki and L. Steg are with the University of Groningen, Grote Kruisstraat 2/1, 9712 TS Groningen, the Netherlands (e-mail: {t.bouman, s.johnson.zawadzki, e.m.steg}@rug.nl).

G. Schuitema is with with the University College Dublin, College of Business, Carysfort Avenue, Blackrock, Dublin, Ireland (e-mail: geertje.schuitema@ucd.ie)

C. Fischione is with the School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology, Stockholm 114 28, Sweden (e-mail: carlofi@kth.se).

in guiding individuals' energy behaviours and choices [14], [15], [16], [17], [18]. These values relate to caring about the environment (i.e., biospheric values), equality and social welfare (i.e., altruistic values), acquiring money, status and possessions (i.e., egoistic values), and acquiring pleasure and comfort (i.e., hedonic values). Typically, individuals endorse all these values to some extent, but differ in how strongly they endorse and prioritize each value, which robustly guide and predict their behaviours and choices. More specifically, individuals are more likely to engage in actions that benefit more prioritized values, and are more likely to refrain from actions that have costs for more prioritized values, and this logic we will incorporate in our models (see Subsection III-D).

More concretely, in this paper, we consider a DC smart grid with a number of prosumers interconnected through resistive-inductive transmission lines. For the considered DC smart grid we propose a novel distributed optimal control scheme that addresses a social-physical welfare problem and allows to share among the prosumers financial, technical and social costs and utilities associated with the generation and consumption of energy, fulfilling (at the steady-state) physical requirements.

To achieve these goals we use an approach that bridges convex optimization and systems theory, i.e., (continuous) primal-dual dynamics [19], [20], [21] and passivity [22]. The contributions of the paper can be summarized as follows:

(1) We formulate a (convex) social-physical welfare optimization problem.
(2) We design a distributed optimal controller, whose (unforced) dynamics represent the primal-dual dynamics of the considered optimization problem.
(3) After showing the passivity properties of the smart grid and the controller, a power-conserving interconnection between the smart grid and the controller is established and the (global) asymptotic convergence of the closed-loop system trajectories to the desired equilibrium point is proved.
(4) The topology of the used communication network can differ from the topology of the physical network.
(5) We provide a social interpretation of some parameters of the control algorithm, focusing on key motives of prosumers and how these relate to the proposed solutions.

The remainder of this paper is outlined as follows. In Section II we describe the model of the considered DC smart grid and show its passivity property. Thereafter, in Section III we state the desired control objectives and, in Section IV we explain the proposed control strategy. We present simulation results in Section V and conclude with some future remarks in Section VI.

A. Notation

The set of real numbers and non-negative real numbers are denoted by $\mathbb{R}$ and $\mathbb{R}_+$, respectively. For a vector $x \in \mathbb{R}^n$ and a symmetric and positive semidefinite matrix $M \in \mathbb{R}^{n\times n}$, let $\|x\|_M := (x^\top M x)^{1/2}$. If M is the identity matrix, this is the Euclidean norm and is denoted by $\|x\|$. For symmetric matrices $P, Q \in \mathbb{R}^{n\times n}$, $P \leq Q$ implies that $Q - P$ is positive semidefinite. $\mathbb{I}$ and $\mathbb{1}$ denote the identity matrix and ones vector of appropriate dimensions, respectively, while $\mathbf{0}$ denote the null matrix (or vector) of appropriate dimensions. Let x, u be the state and input of the physical plant, x^* and u^* denote the corresponding optimization variables. Moreover, $(\overline{u}, \overline{x})$ and $(\overline{u}^*, \overline{x}^*)$ denote the values of (u, x) and (u^*, x^*) at the steady-state, respectively.

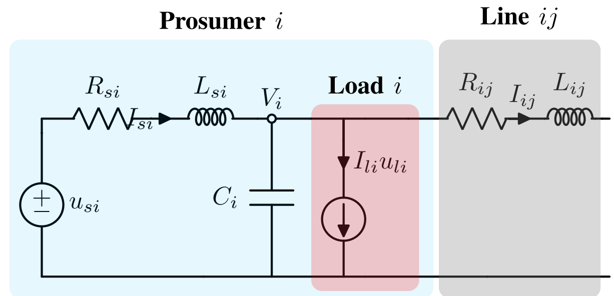

Fig. 1. Electrical scheme of prosumer $i \in \mathcal{V}$ and transmission line $k \sim \{i,j\} \in \mathcal{E}$, $j \in \mathcal{N}_i$, where $\mathcal{N}_i$ is the set of the prosumers connected to prosumer i.

TABLE I
DESCRIPTION OF THE USED SYMBOLS

Symbol	Description
I_{si}	Generated current
R_{si}, L_{si}	Filter resistance, inductance
V_i	Load voltage
C_i	Shunt capacitor
I_k	Line current
R_k, L_k	Line resistance, inductance
u_{si}, u_{li}	Control inputs
I_{li}	Load current

II. DC Smart Grid

In this paper we consider a typical DC smart grid with n prosumers connected to each other through m resistive-inductive (RL) transmission lines. For the readers' convenience, a schematic electrical diagram of the considered smart grid is illustrated in Fig. 1 (see also Table I for the description of the used symbols). Each prosumer is represented by a DC voltage source[1] supplying a controllable load $I_{li}u_{li}$. More precisely, given the load demand I_{li} of prosumer i, the control input $0 \leq u_{li}^{\min} \leq u_{li} \leq 1$ can reduce the demand to $I_{li}u_{li}^{\min}$, which represents the current required to supply the base-loads of prosumer i. The overall DC smart grid is represented by a connected and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the nodes, $\mathcal{V} = \{1, ..., n\}$, represent the prosumers and the edges, $\mathcal{E} = \{1, ..., m\}$, represent the transmission lines interconnecting the prosumers. Therefore, the topology of the smart grid is described by its corresponding incidence matrix $\mathcal{B} \in \mathbb{R}^{n\times m}$. The ends of edge $k \in \mathcal{E}$ are arbitrarily labeled with a $+$ and a $-$, and the entries of $\mathcal{B}$ are given by $\mathcal{B}_{ik} = +1$ if i is the positive end of k, $\mathcal{B}_{ik} = -1$ if i is the negative end of k, and $\mathcal{B}_{ik} = 0$ otherwise. Consequently, the overall

[1]With a slight abuse of nomenclature, the considered DC voltage source can also represent for instance the output voltage of an energy storage system.

dynamical system describing the smart grid behavior can be written compactly for all the prosumers $i \in \mathcal{V}$ as

$$\begin{aligned} L_s \dot{I}_s &= -R_s I_s - V + u_s \\ L\dot{I} &= -RI - \mathcal{B}^\top V \\ C\dot{V} &= I_s + \mathcal{B}I - I_l u_l, \end{aligned} \tag{1}$$

where $I_s, V, u_s, u_l : \mathbb{R}_+ \to \mathbb{R}^n$ and $I : \mathbb{R}_+ \to \mathbb{R}^m$. Moreover, $C, L_s, R_s, I_l \in \mathbb{R}^{n\times n}$ and $R, L \in \mathbb{R}^{m\times m}$ are positive definite diagonal matrices, e.g., $I_l = \operatorname{diag}(I_{l1}, \dots, I_{ln})$. Furthermore, let $x := [I_s^\top, I^\top, V^\top]^\top \in \mathcal{X} \subseteq \mathbb{R}^{2n+m}$ and $u := [u_s^\top, u_l^\top]^\top \in \mathcal{U} \subseteq \mathbb{R}^{2n}$ denote the state and input of system (1), respectively. Then, for a given constant input $\overline{u}$, the corresponding steady state solution $(\overline{I}_s, \overline{I}, \overline{V})$ to system (1) satisfies

$$\overline{V} = -R_s \overline{I}_s + \overline{u}_s \tag{2a}$$
$$\overline{I} = -R^{-1}\mathcal{B}^\top \overline{V} \tag{2b}$$
$$\overline{I}_s = -\mathcal{B}\overline{I} + I_l \overline{u}_l. \tag{2c}$$

Before establishing a useful property of system (1), we first define the set of all feasible forced equilibria of (1) as follows:

$$E := \{(\overline{u}, \overline{x}) \in \mathcal{U} \times \mathcal{X} | (\overline{u}, \overline{x}) \text{ satisfies } (2)\}. \tag{3}$$

Moreover, we observe that for any $u = \overline{u}$, the steady-state solution $\overline{x}$ to (1) is unique and satisfies

$$\begin{aligned} \overline{V} &= (\mathbb{I} + R_s\mathcal{L})^{-1}(\overline{u}_s - R_s I_l \overline{u}_l) \\ \overline{I}_s &= \mathcal{L}(\mathbb{I} + R_s\mathcal{L})^{-1}(\overline{u}_s - R_s I_l \overline{u}_l) + I_l \overline{u}_l \\ \overline{I} &= -R^{-1}\mathcal{B}^\top(\mathbb{I} + R_s\mathcal{L})^{-1}(\overline{u}_s - R_s I_l \overline{u}_l), \end{aligned} \tag{4}$$

where $\mathcal{L} := \mathcal{B}R^{-1}\mathcal{B}^\top$ is the (weighted) Laplacian matrix associated with the physical network.

Now, in analogy with [23], we establish a passivity property of system (1) that will be useful in Section IV for the controller design.

Proposition 1: **(Passivity property of** (1)**)**. Let $y := [\dot{I}_s^\top, -\dot{V}^\top I_l]^\top$ and $u_d := [u_{sd}^\top, u_{ld}^\top]^\top$, $u_{sd}, u_{ld} : \mathbb{R}_+ \to \mathbb{R}^n$. The following statements hold:

(a) System (1) together with $\dot{u} = u_d$ is passive with respect to the supply rate $u_d^\top y$ and storage function

$$S(u, x) = \frac{1}{2}\dot{x}^\top M \dot{x}, \tag{5}$$

with $M := \operatorname{diag}\{L_s, L, C\}$.

(b) Let $u_d = \mathbf{0}$. System (1) converges to the equilibrium point $(\overline{u}, \overline{x}) \in E$.

Proof: The storage function S in (5) satisfies

$$\begin{aligned} \dot{S} &= -\dot{I}_s^\top R_s \dot{I}_s - \dot{I}^\top R \dot{I} + u_{sd}^\top \dot{I}_s - u_{ld}^\top I_l \dot{V} \\ &\le u_{sd}^\top \dot{I}_s - u_{ld}^\top I_l \dot{V} \end{aligned} \tag{6}$$

along the solutions to (1), concluding the proof of part (a). For part (b), we conclude from (6) that there exists a forward invariant set Ω and by LaSalle's invariance principle the solutions that start in Ω converge to the largest invariant set contained in

$$\Omega \cap \left\{(u, x) \in \mathcal{U} \times \mathcal{X} | \dot{u} = \mathbf{0}, \dot{I}_s = \mathbf{0}, \dot{I} = \mathbf{0}\right\}. \tag{7}$$

Moreover, from the first line of (1) it follows that V is also a constant vector in Ω. Then, the solutions that start in Ω converge to the largest invariant set contained in $\Omega \cap E$, concluding the proof of part (b). ■

III. Problem Formulation

In this section, we first introduce an auxiliary (or virtual) state variable that permits to use a communication network whose topology can differ from the one of the electric network. Then, we formulate and discuss the main goal of the paper, which allows to share among the prosumers of the smart grid the financial, technical and social costs and utilities associated with the generation and consumption of energy.

Firstly, we notice that (2) implies the following equalities:

$$\mathbf{0} = \overline{u}_s - R_s \overline{I}_s - \overline{V}, \tag{8a}$$
$$0 = \mathbb{1}^\top \left(\overline{I}_s - I_l \overline{u}_l\right). \tag{8b}$$

Secondly, we observe that the incidence matrix $\mathcal{B}$ satisfies $\mathbb{1}^\top \mathcal{B} = \mathbf{0}$, where $\mathbb{1} \in \mathbb{R}^n$ is the vector consisting of all ones. Therefore, $(\overline{I}_s, \overline{u}_l)$ is a solution to (8b) if and only if there exists $v : \mathbb{R}_+ \to \mathbb{R}^n$ satisfying

$$-I_l \overline{u}_l + \overline{I}_s - \mathcal{L}_c \overline{v} = \mathbf{0}, \tag{9}$$

where $\mathcal{L}_c = \mathcal{B}_c \Gamma \mathcal{B}_c^\top$ denotes the (weighted) Laplacian matrix associated with a connected and undirected communication graph $\mathcal{G}_c = (\mathcal{V}, \mathcal{E}_c)$, $\mathcal{E}_c = \{1, ..., m_c\}$ being the set (possibly different from $\mathcal{E}$) of the communication links between the prosumers of the smart grid. Moreover, $\mathcal{B}_c \in \mathbb{R}^{n\times m_c}$ is the corresponding incidence matrix (defined analogously to $\mathcal{B}$), and $\Gamma \in \mathbb{R}^{m_c \times m_c}$ is a positive definite diagonal matrix describing the weights on the m_c edges.

Now, let $x_a := [I_s^\top, V^\top, v^\top]^\top \in \mathcal{X}_a \subseteq \mathbb{R}^{3n}$. Then, let us define the following set[2]:

$$E_a := \{(\overline{u}, \overline{x}_a) \in \mathcal{U} \times \mathcal{X}_a | (\overline{u}, \overline{x}_a) \text{ satisfies (8a), (9)}\}, \tag{10}$$

which we will use as set of equality constraints of the optimization problem we formulate later in this section.

A. Prosumer's cost and utility

We observe that (8b) implies that at the steady-state the total generated current $\mathbb{1}^\top \overline{I}_s$ is equal to the total current demand $\mathbb{1}^\top I_l \overline{u}_l$. Therefore, there is flexibility to distribute the total required current optimally among the various (equivalent) prosumers. Generally, in order to achieve an efficient demand and supply matching, so avoiding the overstressing of a source, it is desirable that the total load demand of the smart grid is shared among all the prosumers proportionally to the corresponding generation (and/or storage) capacities (*fair* current sharing). This desire is equivalent to achieving $\pi_{ci}\overline{I}_{si} = \pi_{cj}\overline{I}_{sj}$ for all $i, j \in \mathcal{V}$, where a relatively small value of $\pi_{ci} \in \mathbb{R}_+$ corresponds for instance to a relatively large generation (and/or storage) capacity of prosumer i. We call this desire *ideal* current sharing and, in analogy with [12], can be expressed as follows:

$$\lim_{t\to\infty} I_s(t) = \overline{I}_s = \Pi_c^{-1}\mathbb{1} i_s, \tag{11}$$

with $\Pi_c = \operatorname{diag}(\pi_{c1}, \dots, \pi_{cn})$ and $i_s = \mathbb{1}^\top I_l \overline{u}_l / (\mathbb{1}^\top \Pi_c^{-1} \mathbb{1})$. Moreover, a transition towards 100% renewable energy systems requires that end-users change their energy-related behaviours and accept new technologies such as *demand response*, which controls the prosumers' appliances. Therefore,

[2]Note that $(\overline{u}, \overline{I}_s, \overline{V}, \overline{v}) \in E_a \iff (\overline{u}, \overline{I}_s, -R^{-1}\mathcal{B}^\top \overline{V}, \overline{V}) \in E$.

to make the notion of optimality explicit, we assign to every prosumer i a strictly convex quadratic 'cost' function $C_i(I_{si})$ related to the generated current I_{si} and a strictly concave quadratic 'utility' function $U_i(u_{li})$ related to the current consumption $I_{li}u_{li}$, i.e.,

$$C_i(I_{si}) = \frac{1}{2}\pi_{ci}I_{si}^2 \quad \forall i \in \mathcal{V}, \tag{12a}$$

$$U_i(u_{li}) = -\frac{1}{2}\pi_{ui}I_{li}^2\,(1-u_{li})^2 \quad \forall i \in \mathcal{V}, \tag{12b}$$

where a relatively large value of $\pi_{ui} \in \mathbb{R}_+, \forall i \in \mathcal{V}$ corresponds for instance to a relatively large request of comfort from prosumer i. Note that in Subsection III-D we also provide a *social* interpretation of the coefficients π_{ci} and π_{ui} appearing in the cost and utility functions, respectively.

B. Social-physical welfare

Let $C(I_s) := \sum_{i\in\mathcal{V}} C_i(I_{si})$ and $U(u_l) := \sum_{i\in\mathcal{V}} U_i(u_{li})$. Then, we denote the *social welfare* by $W(u_l, I_s) := U(u_l) - C(I_s)$ and consider the following convex minimization problem:

$$\begin{aligned} &\min_{u^*, x_a^*} \; - W\left(u_l^*, I_s^*\right) \\ &\text{s.t. } (\overline{u}^*, \overline{x}_a^*) \in E_a. \end{aligned} \tag{13}$$

Considering the Lagrangian function associated with the optimization problem (13) and manipulating the first-order optimality conditions leads to the following lemma, which makes the solution to (13) explicit:

Lemma 1: **(Optimal social welfare)**. The solution to (13) satisfies

$$\overline{I}_s^* = \Pi_c^{-1}\mathbb{1}\lambda^{\text{opt}} \tag{14a}$$

$$I_l\overline{u}_l^* = \left(I_l - \lambda^{\text{opt}}\Pi_u^{-1}\right)\mathbb{1}, \tag{14b}$$

where

$$\lambda^{\text{opt}} = \frac{\mathbb{1}^\top I_l \mathbb{1}}{\mathbb{1}^\top\left(\Pi_c^{-1} + \Pi_u^{-1}\right)\mathbb{1}}, \tag{15}$$

with $\Pi_u = \text{diag}(\pi_{u1}, \ldots, \pi_{un})$.
Moreover, note that (14a) implies indeed that *ideal* current sharing is achieved (see (11)).

Now, we assume that at the PCC of each prosumer $i \in \mathcal{V}$, there exists a desired voltage:

Assumption 1: **(Desired voltages)**. There exists a desired voltage $V_{di} > 0$ for each $i \in \mathcal{V}$.
However, achieving *ideal* current sharing, prescribes the value of the required differences in voltages among the nodes of the smart grid. As a consequence, it is generally not possible to control the voltage at each node towards the corresponding desired value. For this reason, the voltage requirements are generally relaxed and, as an alternative, several control approaches in the literature propose to regulate the *average* voltage across the whole microgrid towards a global voltage set point [1], [12], where the sources with the largest generation capacity determine the grid voltage, i.e.,

$$\lim_{t\to\infty} \mathbb{1}^\top\Pi_c^{-1}V(t) = \mathbb{1}^\top\Pi_c^{-1}\overline{V} = \mathbb{1}^\top\Pi_c^{-1}V^\star. \tag{16}$$

However, we note that achieving *ideal* current sharing even preserving the average voltage of the smart grid may not always be desired, as it may introduce, in some nodes of the microgrid, large voltage deviations from the corresponding desired value. Consider for instance a DC smart grid with 2 prosumers interconnected through a purely resistive transmission line, the value of which is relatively large (e.g., because the prosumers are spatially distant). Moreover, assume that the load demand of one of the prosumers is much higher than the other. Then, in order to achieve *ideal* current sharing, the prosumers need to share a relatively large current through the transmission line, implying a relatively large voltage deviation (with respect to the desired value) at the corresponding PCCs. Consequently, a steady-state solution satisfying (11) and (16) may be not *feasible* in practical applications. Therefore, in order to address this physical issue we modify (13) as follows:

Objective 1: **(Social-physical welfare)**.

$$\min_{u^*, x_a^*} \; -\alpha W\left(u_l^*, I_s^*\right) + \frac{\beta}{2}\|u_s^*\|^2 + \frac{\gamma}{2}\|V^* - V_d\|^2 \tag{17a}$$

$$\text{s.t. } (\overline{u}^*, \overline{x}_a^*) \in E_a, \tag{17b}$$

where $\alpha, \beta, \gamma \in \mathbb{R}_+$ are design parameters.

Remark 1: **(Rationale behind Objective 1)**. The quadratic function in (17a) comprises three different terms concerning (i) the social welfare, (ii) the control effort and (iii) the voltage deviation from the corresponding desired value. As a consequence, a solution to Objective 1, generally differs from the solution to (13) and does not guarantee the achievement of *ideal* current sharing (11). This leads to a compromise between the social welfare and physical requirements. In order to ensure a proper and safe functioning of the smart grid, the voltage requirement has a priority higher than current sharing. In other words, we are interested in a *feasible* solution that permits to share among the prosumers of the smart grid the largest possible amount of total (controllable) demand in compliance with physical requirements.

C. Additional constraints

In this subsection, we introduce a set of additional inequality constraints, which ensure a safer (steady-state) functioning of the prosumers' appliances and allow prosumer i to choose their acceptable level of flexibility. More precisely, in order to guarantee a proper functioning of the prosumers' appliances, it is generally required that the currents and voltages remain within prescribed limits (see for instance [24] and the references therein). Moreover, we observe from (4) that the steady-state value of the electrical signals are functions of the control inputs u_s, u_l. Therefore, we consider in this paper the following steady-state constraints:

Objective 2: **(Source constraints)**.

$$u_{si}^{\min} \le \lim_{t\to\infty} u_{si}(t) \le u_{si}^{\max}, \tag{18}$$

where $u_{si}^{\min}, u_{si}^{\max} \in \mathbb{R}_+$ denote the minimum and maximum permitted value of the source voltage u_{si}, for all $i \in \mathcal{V}$.

Objective 3: **(Load constraints)**.

$$u_{li}^{\min} \le \lim_{t\to\infty} u_{li}(t) \le 1, \tag{19}$$

where $u_{li}^{\min} \in [0,1)$ denotes the minimum permitted value of u_{li}, for all $i \in \mathcal{V}$. Note also that $1/u_{li}^{\min}$ can be interpreted as the acceptable level of flexibility of prosumer i. Indeed, $u_{li}^{\min}$ close to 0 implies that the acceptable level of flexibility of prosumer i is high, which further implies that prosumer i is willing to reduce is energy demand from I_{li} to $I_{li}u_{li}^{\min}$.

D. Prosumer's cost and utility: a social interpretation

In this subsection, inspired by [14], [15], [16], [17], [18], we provide a social interpretation of the coefficients π_{ci} and π_{ui} appearing in the cost and utility functions (12) of prosumer $i \in \mathcal{V}$. Specifically, for the cost function (12a) we propose the following choice for π_{ci}:

$$\pi_{ci} := \frac{\pi_{ci}^{\text{egoistic}}}{\pi_{ci}^{\text{capacity}} \cdot \pi_{ci}^{\text{altruistic}}} \quad \forall i \in \mathcal{V}, \tag{20}$$

where $\pi_{ci}^{\text{capacity}} \in \mathbb{R}_+$ indicates the size of the generation or storage capacity of prosumer i. Moreover, $\pi_{ci}^{\text{egoistic}} \in \mathbb{R}_+$ indicates how much prosumer i safeguards his/her own resources, while $\pi_{ci}^{\text{altruistic}} \in \mathbb{R}_+$ expresses how much prosumer i cares for fairly sharing. Analogously, for the utility function (12b) we propose the following choice for π_{ui}:

$$\pi_{ui} := \frac{\pi_{ui}^{\text{hedonic}}}{\pi_{ui}^{\text{biospheric}}} \quad \forall i \in \mathcal{V}, \tag{21}$$

where $\pi_{ui}^{\text{hedonic}} \in \mathbb{R}_+$ indicates how much prosumer i seeks the convenience and comfort of controlling one's load oneself, while $\pi_{ui}^{\text{biospheric}} \in \mathbb{R}_+$ expresses how much prosumer i cares about the environment.

IV. Distributed Primal-dual controller

In this section we present a basic primal-dual dynamic controller to achieve Objective 1. Note that, for the sake of exposition and due to the page limitation, we do not include in the following analysis the constraints discussed in Subsection III-C, and refer the interested reader to [25] and the references therein for the theoretical analysis in presence of inequality constraints. Consider the social-physical welfare (17), i.e., Objective 1, and let $\lambda_a, \lambda_b : \mathbb{R}_+ \to \mathbb{R}^n$ denote the Lagrange multipliers corresponding to the constraints (8a) and (9), respectively. Moreover, let $\lambda := [\lambda_a^\top, \lambda_b^\top]^\top$ and $x_c := [u^{*\top}, x_a^{*\top}, \lambda^\top]^\top \in \mathcal{X}_c \subseteq \mathcal{U} \times \mathcal{X}_a \times \mathbb{R}^{2n}$. The Lagrangian function corresponding to the optimization problem (17) is

$$\begin{aligned} \mathcal{L}(x_c) :=& -\alpha W(u_l^*, I_s^*) + \frac{\beta}{2}\|u_s^*\|^2 + \frac{\gamma}{2}\|V^* - V_d\|^2 \\ &+ \lambda_a^\top (u_s^* - R_s I_s^* - V^*) \\ &+ \lambda_b^\top (-I_l u_l^* + I_s^* - \mathcal{L}_c v^*). \end{aligned} \tag{22}$$

Consequently, the first order optimality conditions are given by the Karush-Kuhn-Tucker (KKT) conditions, i.e.,

$$\begin{aligned} \beta \overline{u}_s^* + \overline{\lambda}_a &= \mathbf{0} \\ -\alpha I_l \Pi_u I_l (\mathbb{1} - \overline{u}_l^*) - I_l \overline{\lambda}_b &= \mathbf{0} \\ \alpha \Pi_c \overline{I}_s^* - R_s \overline{\lambda}_a + \overline{\lambda}_b &= \mathbf{0} \\ \gamma(\overline{V}^* - V_d) - \overline{\lambda}_a &= \mathbf{0} \\ -\mathcal{L}_c \overline{\lambda}_b &= \mathbf{0} \\ \overline{u}_s^* - R_s \overline{I}_s^* - \overline{V}^* &= \mathbf{0} \\ -I_l \overline{u}_l^* + \overline{I}_s^* - \mathcal{L}_c \overline{v}^* &= \mathbf{0}. \end{aligned} \tag{23}$$

Moreover, we notice that the optimization problem (17) is convex and the feasibility set E_a is nonempty. As a consequence, the optimization problem satisfies the Slater's condition and, therefore, strong duality holds [26]. Hence, $\overline{u}_s^*, \overline{u}_l^*, \overline{I}_s^*, \overline{V}^*, \overline{v}^*$ are optimal if and only if there exist $\overline{\lambda}_a, \overline{\lambda}_b$ satisfying (23).

Now, consider the following dynamic controller, designed using the primal-dual dynamics of the optimization problem (17):

$$-\tau_s \dot{u}_s^* = \beta u_s^* + \lambda_a - \nu_s \tag{24a}$$
$$-\tau_l \dot{u}_l^* = -\alpha I_l \Pi_u I_l (\mathbb{1} - u_l^*) - I_l \lambda_b - \nu_l \tag{24b}$$
$$-\tau_I \dot{I}_s^* = \alpha \Pi_c I_s^* - R_s \lambda_a + \lambda_b \tag{24c}$$
$$-\tau_V \dot{V}^* = \gamma(V^* - V_d) - \lambda_a \tag{24d}$$
$$-\tau_v \dot{v}^* = -\mathcal{L}_c \lambda_b \tag{24e}$$
$$\tau_a \dot{\lambda}_a = u_s^* - R_s I_s^* - V^* \tag{24f}$$
$$\tau_b \dot{\lambda}_b = -I_l u_l^* + I_s^* - \mathcal{L}_c v^*, \tag{24g}$$

where $\tau_s, \tau_l, \tau_I, \tau_V, \tau_v, \tau_a, \tau_b > 0$ are design parameters and $\nu_s, \nu_l : \mathbb{R}_+ \to \mathbb{R}^n$ denote the controller input ports, which will be used later to interconnect the controller (24) with the plant (1). Let $\nu := [\nu_s^\top, \nu_l^\top]^\top$. Then, we define the forced equilibria set of system (24) as follows:

$$E_c := \left\{ (\overline{x}_c, \overline{\nu}) \in \mathcal{X}_c \times \mathbb{R}^{2n} | \dot{x}_c = \mathbf{0} \right\}. \tag{25}$$

Moreover, for any $\nu = \overline{\nu}$, a tedious but straightforward calculation permits to prove[3] the existence and uniqueness of the solution $\overline{x}_c$ to (24). Then, we establish a passivity property of the controller (24) that will be useful later in this section for ensuring the stability of the closed-loop system.

Proposition 2: **(Passivity property of** (24)**)** Let $y := [\dot{u}_s^{*\top}, \dot{u}_l^{*\top}]^\top$ and $\nu_d := [\nu_{sd}^\top, \nu_{ld}^\top]^\top$, $\nu_{sd}, \nu_{ld} : \mathbb{R}_+ \to \mathbb{R}^n$. The following statements hold:

(a) The primal-dual controller (24) with $\dot{\nu} = \nu_d$ is passive with respect to the supply rate $\nu_d^\top y_c$ and storage function

$$S_c(x_c, \nu) = \frac{1}{2} \dot{x}_c^\top \tau \dot{x}_c, \tag{26}$$

with $\tau := \text{blockdiag}\{\tau_s, \tau_l, \tau_I, \tau_V, \tau_v, \tau_a, \tau_b\}$.

(b) Let $\nu_d = \mathbf{0}$. The primal-dual controller (24) converges to the equilibrium point $(\overline{x}_c, \overline{\nu}) \in E_c$.

[3] To prove the result we use the additional equality $\mathbb{1}^\top v^*(t) = \mathbb{1}^\top v^*(0)$, for all $t \geq 0$, which follows from (24e).

Proof: The storage function S_c in (26) satisfies

$$\begin{aligned}\dot{S}_c &= -\beta \dot{u}_s^{*\top}\dot{u}_s^* - \alpha \dot{u}_l^{*\top} I_l \Pi_u I_l \dot{u}_l^* - \dot{I}_s^{*\top} R_s \dot{I}_s^* \\ &\quad - \gamma \dot{V}^{*\top}\dot{V}^* + \nu_{sd}^\top \dot{u}_s^* + \nu_{ld}^\top \dot{u}_l^* \\ &\leq \nu_{sd}^\top \dot{u}_s^* + \nu_{ld}^\top \dot{u}_l^*,\end{aligned} \tag{27}$$

along the solutions to (24), concluding the proof of part (a). For part (b), we conclude from (27) that there exists a forward invariant set Ω and by LaSalle's invariance principle the solutions that start in Ω converge to the largest invariant set contained in

$$\Omega \cap \left\{(x_c, \nu) \in \mathcal{X}_c \times \mathbb{R}^{2n} | \dot{u}^* = \mathbf{0}, \dot{I}_s^* = \mathbf{0}, \dot{V}^* = \mathbf{0}, \dot{\nu} = \mathbf{0}\right\}. \tag{28}$$

Moreover, from (24a) and (24b) it follows that λ_a and λ_b are also constant vectors in Ω. Furthermore, from (24e) and (24g) it follows that $\mathbb{1}^\top \dot{v}^* = 0$ and $\mathcal{L}_c \dot{v}^* = \mathbf{0}$, respectively, implying that v^* is also a constant vector in Ω. Then, the solutions that start in Ω converge to the largest invariant set contained in $\Omega \cap E_c$, concluding the proof of part (b). ■

Now, the passive dynamic controller (24) is interconnected to the physical grid (1) by choosing $u = u^*$, $\nu_s = -I_s$ and $\nu_l = I_l V$. Consequently, we obtain the following closed-loop system:

$$L_s \dot{I}_s = -R_s I_s - V + u_s^* \tag{29a}$$
$$L \dot{I} = -RI - \mathcal{B}^\top V \tag{29b}$$
$$C \dot{V} = I_s + \mathcal{B} I - I_l u_l^* \tag{29c}$$
$$-\tau_s \dot{u}_s^* = I_s + \beta u_s^* + \lambda_a \tag{29d}$$
$$-\tau_l \dot{u}_l^* = -I_l V - \alpha I_l \Pi_u I_l (\mathbb{1} - u_l^*) - I_l \lambda_b \tag{29e}$$
$$-\tau_I \dot{I}_s^* = \alpha \Pi_c I_s^* - R_s \lambda_a + \lambda_b \tag{29f}$$
$$-\tau_V \dot{V}^* = \gamma (V^* - V_d) - \lambda_a \tag{29g}$$
$$-\tau_v \dot{v}^* = -\mathcal{L}_c \lambda_b \tag{29h}$$
$$\tau_a \dot{\lambda}_a = u_s^* - R_s I_s^* - V^* \tag{29i}$$
$$\tau_b \dot{\lambda}_b = -I_l u_l^* + I_s^* - \mathcal{L}_c v^*. \tag{29j}$$

The set of all feasible operating points of (29) is defined as

$$\begin{aligned}E_{cl} := \{\, &(\overline{x}, \overline{x}_c) \in \mathcal{X} \times \mathcal{X}_c | \\ &(\overline{u}^*, \overline{x}) \in E, (\overline{x}_c, -\overline{I}_s, I_l \overline{V}) \in E_c\}.\end{aligned} \tag{30}$$

Remark 2: **(A perhaps surprising additional penalty)**. Note that the steady-state conditions of (24) represent the KKT conditions of the optimization problem (17) with the additional penalty $-\overline{\nu}_s^\top u_s^* - \overline{\nu}_l^\top u_l^*$, which becomes $\overline{I}_s^\top u_s - \overline{V} I_l^\top u_l$ after interconnecting (24) with (1). Then, from (2) we obtain the following expression for the additional penalty:

$$\overline{I}_s^\top u_s - \overline{V} I_l^\top u_l = \overline{I}_s^\top R_s \overline{I}_s + \overline{V}^\top \mathcal{B} R^{-1} \mathcal{B}^\top \overline{V},$$

which implies that also the total power losses in the filters and transmission lines are penalized.

We can now present the main result of the paper, i.e., the closed-loop stability.

Proposition 3: **(Stability)**. Let Assumption 1 hold and assume E_{cl} in (30) to be nonempty. The closed-loop system (29) stabilizes to the operating point $(\overline{x}, \overline{x}_c) \in E_{cl}$.

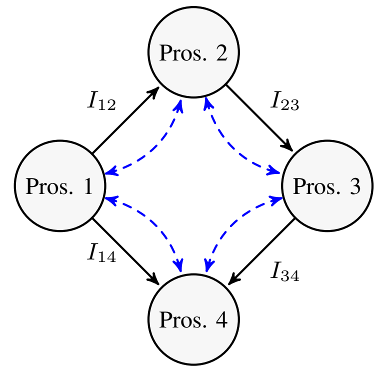

Fig. 2. Scheme of the considered smart grid with 4 prosumers (Pros.). The solid arrows indicate the positive direction of the current flows through the power network, while the dashed lines represent the communication network.

Proof: The storage function

$$S_{cl}(x, x_c) := S(u^*, x) + S_c(x_c, -I_s, I_l V) \tag{31}$$

satisfies

$$\begin{aligned}\dot{S}_{cl} = &- \dot{I}_s^\top R_s \dot{I}_s - \dot{I}^\top R \dot{I} - \beta \dot{u}_s^{*\top} \dot{u}_s^* \\ &- \alpha \dot{u}_l^{*\top} I_l \Pi_u I_l \dot{u}_l^* - \dot{I}_s^{*\top} R_s \dot{I}_s^* - \gamma \dot{V}^{*\top} \dot{V}^*\end{aligned} \tag{32}$$

along the solutions to the closed-loop system (29). Therefore, there exists a forward invariant set Ω and by LaSalle's invariance principle the solutions that start in Ω converge to the largest invariant set contained in

$$\begin{aligned}\Omega \cap \{(x, x_c) \in \mathcal{X} \times \mathcal{X}_c | \dot{I}_s = \mathbf{0}, \dot{I} = \mathbf{0}, \\ \dot{u}^* = \mathbf{0}, \dot{I}_s^* = \mathbf{0}, \dot{V}^* = \mathbf{0}, \dot{\nu} = \mathbf{0}\}.\end{aligned} \tag{33}$$

Then, from the proofs of Propositions 1 and 2, we conclude that the solutions starting in Ω converge to the largest invariant set contained in $\Omega \cap E_{cl}$, concluding the proof of the proposition. ■

V. SIMULATION RESULTS

In this section, the proposed distributed optimal primal-dual controller (24) is assessed in simulation, by implementing a smart grid comprising four prosumers (Pros.) connected as illustrated in Figure 2. Three different scenarios are investigated and discussed. The physical parameters of the smart grid are reported in Table II, while the social parameters π_{ci} and π_{ui} appearing in the cost and utility functions of prosumer i are reported in Table III. Moreover the acceptable level of flexibility of prosumer i in (19) is chosen as $u_{li}^{\min} = 0.4$, for each prosumer of the considered smart grid. All the τ-parameters of the proposed primal-dual controller (24) are selected equal to 1, and in the optimization problem (17) we choose $\alpha = \beta = \gamma = 1$. Before discussing the three considered scenarios in the following, we notice that for negligible values of the parasitic resistance R_{si}, the inequality constraint (18) implies $V_i^{\min} \leq \overline{V}_i \leq V_i^{\max}$. We select $V_{di} = 380\,\text{V}$, $V_i^{\min} = 378\,\text{V}$ and $V_i^{\max} = 382\,\text{V}$ for all the prosumers.

In the first scenario we consider a realistic scenario in which all the prosumers have different but homogeneous behaviors. We can observe in Figure 3 that the voltage at the PCC

TABLE II
PHYSICAL PARAMETERS

		Pros. 1	Pros. 2	Pros. 3	Pros. 4
L_{si}	(mH)	1.8	2.0	3.0	2.2
C_{si}	(mF)	2.2	1.9	2.5	1.7
R_{si}	(mΩ)	2.0	3.0	1.5	1.0
I_{li}	(A)	30.0	30.0	30.0	30.0
$\pi_{ci}^{\text{capacity}}$		1	1	1	1
		Line 1	Line 2	Line 3	Line 4
R_k	(mΩ)	70	50	80	60
L_k	(μH)	2.1	2.0	3.0	2.2

TABLE III
SOCIAL PARAMETERS

Scenario		Pros. 1	Pros. 2	Pros. 3	Pros. 4
1	π_{ci}	$5/8$	$6/7$	$7/6$	$8/5$
	π_{ui}	$8/7$	$9/6$	$9/5$	$8/3$
2	π_{ci}	$50/8$	$60/7$	$70/6$	$80/5$
	π_{ui}	$8/7$	$9/6$	$9/5$	$8/3$
3	π_{ci}	$5/8$	$6/7$	$7/6$	$80/5$
	π_{ui}	$8/7$	$9/6$	$9/5$	$8/3$

of each prosumer remains within the bounds that ensure a proper functioning of the connected appliances and a safe and reliable functioning of the overall grid. Moreover, the voltage difference between the PCCs of the prosumers imply that they are sharing current flows. In particular, we can observe that the most selfish and less altruistic prosumer (Pros. 4) is generating less current than for instance the least selfish and most altruistic prosumer (Pros. 1). We can also observe that the prosumers who care more about the environment and less about the comfort reduce the current demand to lower values than prosumers who care more about comfort and less about environment.

In the second scenario we investigate the case in which all the prosumers are much more selfish. From Figure 4 we can observe that the disadvantage for not cooperating and sharing the generated currents is the maximum permitted reduction of the load demand, implying less comfort and pleasure for all the prosumers.

To conclude, in the last scenario we investigate the case in which only one prosumer (Pros. 4) is defecting from the cooperation strategy. Indeed, we consider Pros. 4 to be much more selfish than all the other prosumers of the considered smart grid. From Figure 5 we observe that Pros. 4 is taking advantage of the altruistic prosumers. This scenario opens interesting future research directions towards the inclusion of price-based mechanisms and/or the analysis of imitation dynamics in social networks.

VI. CONCLUSION

Our work shows how incorporating social aspects of prosumers in a DC smart grid can affect the outcomes and performance of the proposed primal-dual controller. This not only provides initial insights in how social aspects could be integrated in control algorithms and how this could be of use, but also identifies key issues future research should address, resulting in the following research agenda. Firstly, future

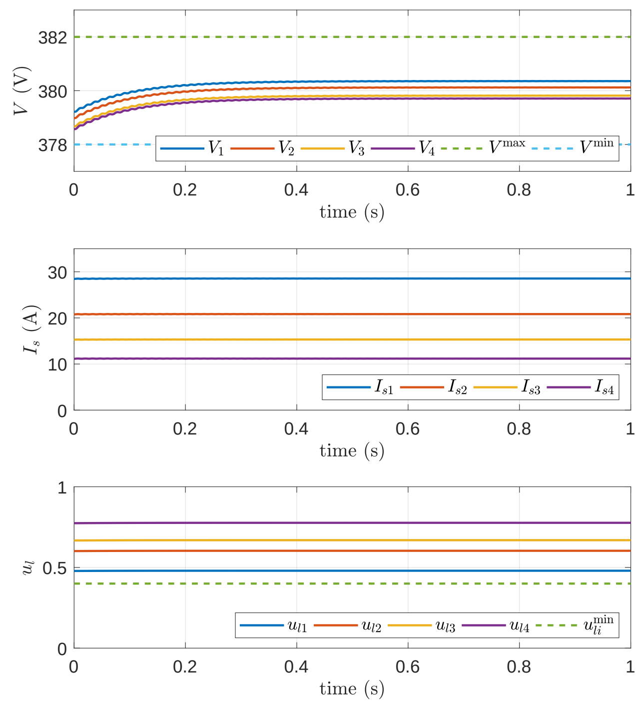

Fig. 3. Scenario 1. From the top: voltage at the PCC; generated current; load control input.

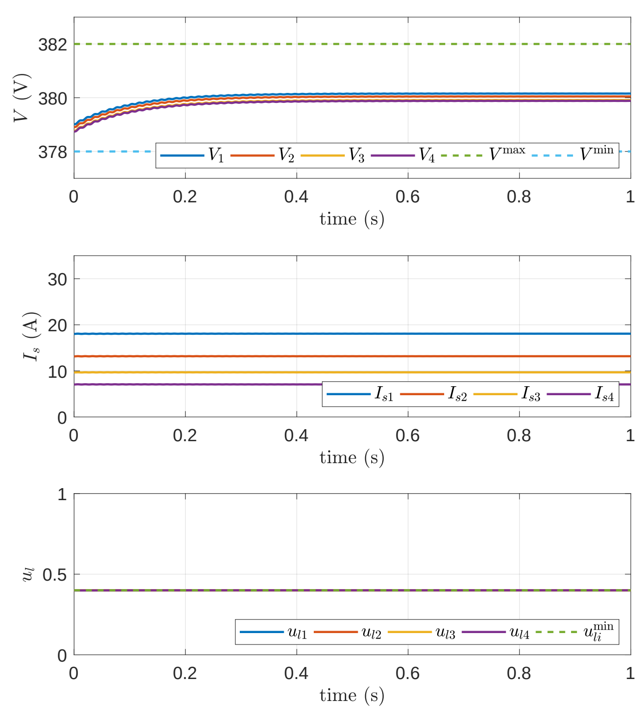

Fig. 4. Scenario 2. From the top: voltage at the PCC; generated current; load control input.

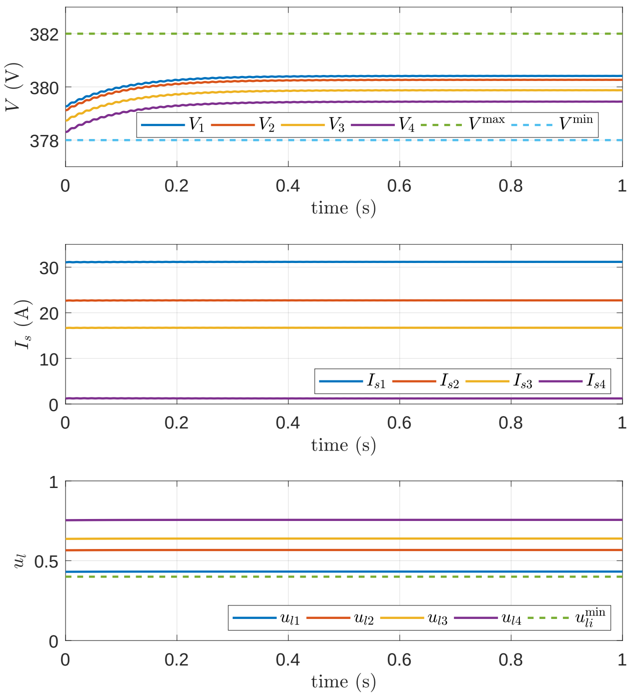

Fig. 5. Scenario 3. From the top: voltage at the PCC; generated current; load control input.

research should provide insights in which numerical values are associated with the value-based parameters (i.e., egoistic, altruistic, hedonic and biospheric). Secondly, research should focus on the dynamics of social aspects over time. For example, how will prosumers react when one prosumer defects, for instance because this person has strong egoistic values (Scenario 3). One option might be that the defector might start sharing power because of social pressure and norms; alternatively, the defector might demotivate others to share their generated power, reducing the amount of power shared within the local network and resulting in an uncomfortable solution (Scenario 2). Working on these questions, for which our paper provides initial steps, could greatly contribute to a transition towards 100% renewable energy systems, making solution more acceptable, desirable and realistic.

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