A Consensus-Based Controller for DC Power Networks ⋆

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Abstract: In this paper we propose a new distributed passivity-based control scheme, achieving proportional (fair) current sharing and average voltage regulation in Direct Current (DC) power networks, with an arbitrary topology. The considered DC network is composed of several Distributed Generation Units (DGUs) interconnected through resistive-inductive power lines. Each DGU includes a generic energy source that supplies an unknown constant impedance load through a DC-DC buck converter. The proposed distributed control scheme achieves current sharing and average voltage regulation, independently of the initial condition of the controlled network, facilitating Plug-and-Play capabilities. Moreover, the proposed control strategy exploits a communication network to achieve current sharing using a consensus-like protocol. Global convergence to a desired steady state is proven and simulations show satisfactory performance.

Keywords: Complex systems, Distributed control, Electrical networks, Current sharing, Voltage control.

1. INTRODUCTION

Nowadays, the most relevant challenge in power networks deals with the integration of smaller and Distributed Generation Units (DGUs), typically from renewable energy sources. Low-voltage electrical distribution networks composed of clusters of DGUs and loads interconnected through power lines are called *microgrids* (Lasseter and Paigi, 2004). Due to historical reasons, in the last decade research mainly focused on the control of Alternate Current (AC) networks (Trip et al., 2014; Cucuzzella et al., 2017a; Weitenberg and De Persis, 2018; Guerrero et al., 2011). However, nowadays several sources and loads (e.g. photovoltaic panels, energy storage systems, electronic appliances) can be directly connected to a Direct Current (DC) network by using DC-DC power converters. Furthermore, due to the reduction of lossy DC-AC conversion stages and the absence of frequency and reactive power control, DC networks appear more efficient and reliable than AC networks (Justo et al., 2013). For these, and more, reasons, control of DC microgrids recently gained growing interest.

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1.1 Literature review

Two common control objectives in DC microgrids are voltage regulation and current sharing or, equivalently, load sharing. The first objective is required to ensure a proper functioning of connected loads (Cucuzzella et al., 2018a, 2017b; Sadabadi et al., 2017; Jeltsema and Scherpen, 2004; Dragičević et al., 2016), while the second one allows the DGUs to share the total network demand proportionally to their generation capacity. This prevents indeed to overstress any source preserving the safety of the network (Beerten and Belmans, 2013). Conventionally, in order to achieve both objectives, hierarchical control schemes are proposed, often exploiting a communication network (Guerrero et al., 2011). As a consequence, scalability of possible control schemes is required to make the control synthesis simple and independent of the knowledge of the whole microgrid. This motivated a growing interest in the development of distributed controllers, particularly aiming at current (load) sharing (Nasirian et al., 2015; Zhao and Dörfler, 2015; Tucci et al., 2018; Cucuzzella et al., 2018b; De Persis et al., 2018; Han et al., 2018). However, the requirement of current sharing does generally not permit to regulate the voltage at each node towards its corresponding nominal value. As a consequence, a reasonable alternative goal is to regulate the average voltage across the whole microgrid (not at a specific node) towards a global prescribed voltage level (e.g., the average of the voltage references). In the literature, this kind of voltage regulation is called 'average voltage regulation', 'global voltage regulation' or 'voltage balancing' (Nasirian et al., 2015; Tucci et al., 2018).

1.2 Main contributions

In this work we design a novel passivity-based control (PBC) scheme that provably guarantees current sharing and average voltage regulation for DC networks comprising buck converters and constant impedance loads. This is achieved by using a standard consensus protocol and by injecting damping (virtual resistors) to modify the dissipation structure of the power network. We outline now the main contributions of this work together with a brief comparison with existing theoretical results considering both the aforementioned control objectives:

1) Although the considered network model is fairly standard, the presented results take particularly into account a possible meshed network topology, incorporating dynamic resistive-inductive lines, which are neglected in e.g. Tucci et al. (2018) and De Persis et al. (2018), where purely resistive lines are considered.

2) The proposed control scheme is distributed and only local measurements of voltage and generated current are needed, as well as informations on the currents generated by the neighbouring DGUs, exploiting a communication network. Moreover, the topology of the communication network is designed independently from the topology of the *physical* network, in contrast to the results provided in Tucci et al. (2018), where an assumption is introduced on the product between the Laplacian matrices associated to the physical and communication networks (Tucci et al., 2018, Assumption 4). Finally, a rule based on classical Brayton-Moser equations is provided to tune the values of the virtual resistors injected by the proposed PBC controllers. As a consequence, the control synthesis is simpler than e.g. the one proposed in Tucci et al. (2018), which requires to solve a Linear Matrix Inequality (LMI) problem for each local primary controller.

3) Global convergence to a desired steady state is guaranteed, independently from the initial condition of the physical power network and the controller state. This is in contrast to e.g. De Persis et al. (2018), where a suitable initialization of the voltages is assumed, or Tucci et al. (2018) and Cucuzzella et al. (2018b) where a suitable initialization of the controller state is required.

1.3 Outline

The remainder of this paper is organized as follows. The network model is presented in Section 2, while the control problem is formulated in Section 3. In Section 4, the proposed PBC control scheme is introduced, whereafter the stability of the controlled network is studied in Section 5. In Section 6, the simulation results are illustrated and discussed, and finally, conclusions are gathered in Section 7.

2. DC NETWORK MODEL

In this paper we study a typical DC network composed of n Distributed Generation Units (DGUs) connected to each other through m resistive-inductive (RL) power lines. A schematic electrical diagram of the considered DC network including a DGU and a power line is represented in Fig. 1 (see also Table 1 for the description of the used symbols).

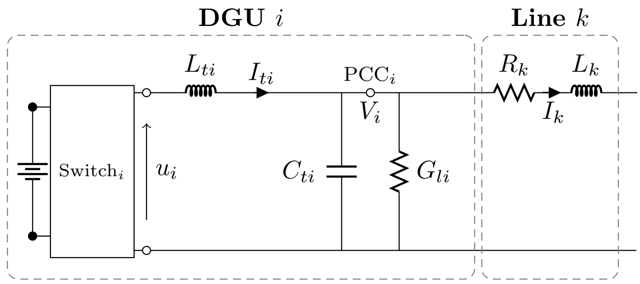

Fig. 1. Electrical scheme of DGU i and line k.

Table 1. Description of the used symbols

Symbol	Description
I_{ti}	Generated current
V_i	Load voltage
I_k	Line current
L_{ti}	Filter inductance
C_{ti}	Shunt capacitor
G_{li}	Load conductance
R_k	Line resistance
L_k	Line inductance
u_i	Control input

The energy source of each DGU is represented by a DC voltage source that supplies a local load through a DC-DC buck converter equipped with an output low-pass filter L_tC_t. The local DC load is connected to the so-called Point of Common Coupling (PCC). By exploiting the Kirchhoff's current (KCL) and voltage (KVL) laws, the equations describing the dynamic behaviour of the DGU i are given by

$$\begin{aligned} L_{ti}\dot{I}_{ti} &= -V_i + u_i \\ C_{ti}\dot{V}_i &= I_{ti} - G_{li}V_i - \sum_{k\in\mathcal{E}_i} I_k, \end{aligned} \tag{1}$$

where $\mathcal{E}_i$ is the set of power lines incident to the DGU i, while the control input u_i represents the buck converter output voltage [1]. The current from DGU i to DGU j is denoted by I_k, and its dynamic is given by

$$L_k\dot{I}_k = (V_i - V_j) - R_kI_k. \tag{2}$$

The symbols used in (1) and (2) are described in Table 1.

The overall DC power network is represented by a connected and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the nodes, $\mathcal{V} = \{1, ..., n\}$, represent the DGUs and the edges, $\mathcal{E} = \{1, ..., m\}$, represent the power lines interconnecting the DGUs. The network topology is described by its corresponding incidence matrix $\mathcal{B} \in \mathbb{R}^{n\times m}$. The ends of edge k are arbitrarily labeled with a $+$ and a $-$, and the entries of $\mathcal{B}$ are given by

$$\mathcal{B}_{ik} = \begin{cases} +1 & \text{if } i \text{ is the positive end of } k \\ -1 & \text{if } i \text{ is the negative end of } k \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Consequently, the overall system can be written compactly for all DGUs $i \in \mathcal{V}$ as

[1] Note that u_i in (1) can be expressed as $\delta_i V_{DC_i}$, where δ_i is the duty cycle of the buck i and V_{DC_i} is the DC voltage source provided by a generic energy source at node i.

$$L_t \dot{I}_t = -V + u$$
$$L\dot{I} = -RI - \mathcal{B}^T V \quad (4)$$
$$C_t \dot{V} = I_t + \mathcal{B}I - G_l V,$$

where $I_t, V, u \in \mathbb{R}^n$, and $I \in \mathbb{R}^m$. Moreover, $C_t, L_t, G_l \in \mathbb{R}^{n\times n}$ and $R, L \in \mathbb{R}^{m\times m}$ are positive definite diagonal matrices, e.g., $C_t = \mathrm{diag}(C_{t1}, \ldots, C_{tn})$.

Remark 1. (**Kron reduction**). Note that in (1), the load currents are located at the PCC of each DGU (see also Figure 1). This situation is generally obtained by a Kron reduction of the original network, yielding an equivalent representation of the network (Zhao and Dörfler, 2015).

3. PROBLEM FORMULATION

In this section we formulate two common control objectives in DC networks. First, we notice that for given constant inputs $\overline{u}$, a steady state solution $(\overline{I}_t, \overline{I}, \overline{V})$ to system (4) satisfies

$$\overline{V} = \overline{u} \quad (5a)$$
$$\overline{I} = -R^{-1}\mathcal{B}^T\overline{V} \quad (5b)$$
$$G_l\overline{V} - \overline{I}_t = \mathcal{B}\overline{I}. \quad (5c)$$

Equation (5c) implies[2] that at the steady state the total generated current $\mathbb{1}^T\overline{I}_t$ is equal to the total current demand $\mathbb{1}^T G_l\overline{V}$. To avoid the overstressing of a source and to improve the generation efficiency, it is generally desired that the total demand of the network is shared among all the various DGUs proportionally to the generation capacity of their corresponding energy sources (proportional current sharing). This desire can be formulated as $w_i\overline{I}_{ti} = w_j\overline{I}_{tj}$ for all $i, j \in \mathcal{V}$, where a relatively large value of w_i corresponds to a relatively small generation capacity of DGU i. This leads to the first control objective concerned with the steady state value of the generated currents $\overline{I}_t$.

Objective 1. (**Current sharing**).

$$\lim_{t\to\infty} I_t(t) = \overline{I}_t = W^{-1}\mathbb{1}i_t^*, \; i_t^* \in \mathbb{R}, \quad (6)$$

with $W = \mathrm{diag}(w_1, \ldots, w_n)$, $w_i > 0$, for all $i \in \mathcal{V}$ and $i_t^* = \mathbb{1}^T G_l\overline{u}/(\mathbb{1}^T W^{-1}\mathbb{1})$.

Note that the steady state requirement $\mathbb{1}^T\overline{I}_t = \mathbb{1}^T G_l\overline{V}$ necessarily prescribes that $i_t^* = \mathbb{1}^T G_l\overline{u}/(\mathbb{1}^T W^{-1}\mathbb{1})$. Before introducing the second control objective considered in this work, we assume that for every DGU i, there exists a nominal reference voltage $V_i^\star$.

Assumption 1. (**Nominal voltages**). There exists a reference voltage[3] $V_i^\star$ at the PCC, for all $i \in \mathcal{V}$.

Generally, achieving Objective 1 does not permit a steady state voltage $\overline{V} = V^\star$. We therefore aim at an *average voltage regulation*, where the weighted average value of $\overline{V}$ is identical to the weighted average value of the nominal voltages $V^\star$. Following the standard practise where the sources with the largest generation capacity determine the grid voltage, we select a weight of $1/w_i$ for all $i \in \mathcal{V}$, leading to the second objective.

Objective 2. (**Average voltage regulation**).

$$\lim_{t\to\infty} \mathbb{1}^T W^{-1}V(t) = \mathbb{1}^T W^{-1}\overline{V} = \mathbb{1}^T W^{-1}V^\star. \quad (7)$$

4. PROPOSED SOLUTION

Before proposing a distributed controller achieving the objectives discussed in the previous section, we make the following assumption on the available informations:

Assumption 2. (**Available informations**). The voltage V_i and the current I_{ti} are measurable at DGU $i \in \mathcal{V}$. The filter inductance L_{ti} is available.

To permit the design of a distributed controller achieving Objective 1, we need that each DGU exchanges informations with its neighbouring DGUs exploiting a communication network, which satisfies the following assumption:

Assumption 3. (**Communication network**). The graph $\mathcal{G}^c = (\mathcal{V}, \mathcal{E}^c)$ corresponding to the topology of the communication network is undirected and connected, where $\mathcal{E}^c = \{1, ..., m_c\}$ represents the set of the communication links between the DGUs[4].

Then, we describe the communication network topology by defining the corresponding incidence matrix $\mathcal{B}^c \in \mathbb{R}^{n\times m_c}$, similarly to $\mathcal{B}$ in (3).

Now, we augment system (4) with additional state variables (distributed integrators) θ_i, $i \in \mathcal{V}$, with dynamics given by

$$\dot{\theta}_i = \sum_{j\in\mathcal{N}_i^c} \gamma_{ij}(w_i I_{ti} - w_j I_{tj}), \quad (8)$$

where $\mathcal{N}_i^c$ is the set of the DGUs that communicate with the DGU i, and $\mathcal{L}^c = \mathcal{B}^c\Gamma(\mathcal{B}^c)^T$ is the (weighted) Laplacian matrix associated to the communication network. The matrix $\Gamma \in \mathbb{R}^{m_c\times m_c}$ is positive definite, diagonal and its entries $\gamma_{ij} = \gamma_{ji} \in \mathbb{R}_{>0}$ describe the edge weights. Then, the dynamics in (8) can be expressed compactly for all nodes $i \in \mathcal{V}$ as

$$\dot{\theta} = \mathcal{L}^c W I_t, \quad (9)$$

that indeed has the form of a consensus protocol, permitting a steady state where $W\overline{I}_t \in \mathrm{im}(\mathbb{1})$ (see also Objective 1).

Following the procedure suggested in Jeltsema and Scherpen (2004), we can now design a passivity-based[5] control scheme for system (4) augmented with (9). To do this, let $\xi = [\xi_1, \xi_2, \xi_3, \xi_4]^T$ be the desired trajectories for the state $x = [I_t, I, V, \theta]^T$ of system (4), (9). Furthermore, we provisionally assume that all the model parameters are perfectly known, and a steady state solution $(\overline{I}_t, \overline{I}, \overline{V}, \overline{\theta})$ to system (4), (9), achieving Objective 1 and Objective 2, exists and is known as well. Note that we make this

[2] The incidence matrix $\mathcal{B}$, satisfies $\mathbb{1}^T\mathcal{B} = \mathbf{0}$, where $\mathbb{1} \in \mathbb{R}^n$ is the vector consisting of all ones.

[3] Often the values for $V_i^\star$ are chosen identical for all $i \in \mathcal{V}$. However, the control strategy proposed later in this work permits to select also non-identical values for $V_i^\star$.

[4] Note that the topology of the communication network can differ from the topology of the physical network.

[5] The rationale behind the design of a passivity-based control is to modify the closed-loop co-energy and inject damping by modifying the dissipative structure of the system (Ortega et al., 2013).

provisional assumption only for the purpose of analysis. The controller we design in the remainder of this section does not require more information than available according to Assumption 2 (see also Remark 3). Then, we make a copy of system (4), (9) in terms of ξ, and inject damping, i.e.,

$$L_t\dot{\xi}_1 = -\xi_3 + W\mathcal{L}^c(\theta - \xi_4) + u \tag{10a}$$
$$L\dot{\xi}_2 = -R\xi_2 - \mathcal{B}^T\xi_3 \tag{10b}$$
$$C_t\dot{\xi}_3 = \xi_1 + \mathcal{B}\xi_2 - G_l\xi_3 + G_a(V - \xi_3) \tag{10c}$$
$$\dot{\xi}_4 = \mathcal{L}^c W\xi_1, \tag{10d}$$

where the term $W\mathcal{L}^c(\theta - \xi_4)$ in (10a) is added to have a suitable interconnection with controller state θ (see (Trip et al., 2019)), evolving according to (9). Furthermore, $G_a \in \mathbb{R}^{n\times n}$ in (10c) is a positive definite diagonal matrix, and its entries are *virtual* resistors connected in parallel to the *real* capacitors and load resistors of the power network (4).

We now explicitly define the desired trajectories ξ for the state of system (10):

$$\xi_1(t) = W^{-1}\mathbb{1}\alpha(t),\ \alpha \in \mathbb{R} \tag{11a}$$
$$\xi_2 = -R^{-1}\mathcal{B}^T\xi_3 \tag{11b}$$
$$\xi_3 = V^\star - W\mathcal{L}^c\xi_4 \tag{11c}$$
$$\dot{\xi}_4 = \mathbf{0}, \tag{11d}$$

where (11a) is needed[6] to achieve proportional current sharing (see Objective 1). Then, (11d) follows from (10d) together with (11a), and it implies that $\xi_4(t) = \xi_4(0)$ for any $t \geq 0$. Furthermore, we impose $\xi_4(0) = \overline{\theta}$, and select as desired voltage $\xi_3 = V^\star - W\mathcal{L}^c\overline{\theta}$, which is constant. As a consequence, $\xi_2 = -R^{-1}\mathcal{B}^T\xi_3$.

Finally, an explicit definition of the control action is obtained after solving system (10) for u. To do so, we first compute the derivative with respect to time of (10c), yielding:

$$\mathbf{0} = \dot{\xi}_1 + G_a\dot{V}, \tag{12}$$

where we have exploited that ξ_2 and ξ_3 are constant. Then, after substituting in (12) the dynamics of ξ_1 given by (10a), and the desired trajectories (11), we obtain

$$u = -L_tG_a\dot{V} - W\mathcal{L}^c\theta + V^\star. \tag{13}$$

Although the first time derivative of the voltage at PCC is not available (see Assumption 2), we rely, for instance, on the well-known Levant's differentiator (Levant, 2003) to retrieve $\dot{V}$ in a finite time, using only the measure of V.

System (4), (9), interconnected with the distributed controller (13), yields the overall closed-loop system

$$L_t\dot{I}_t = -V - L_tG_a\dot{V} - W\mathcal{L}^c\theta + V^\star \tag{14a}$$
$$L\dot{I} = -RI - \mathcal{B}^TV \tag{14b}$$
$$C_t\dot{V} = I_t + \mathcal{B}I - G_lV \tag{14c}$$
$$\dot{\theta} = \mathcal{L}^cWI_t, \tag{14d}$$

where θ_i in (14a) plays the role of modifying the nominal voltage $V_i^\star$ at each PCC in order to achieve current sharing (Objective 1), while guaranteeing average voltage regulation at steady state. Indeed, notice that after premultiplying both sides of (14a) with $\mathbb{1}^TW^{-1}$, realizing that at steady state $\dot{V} = \mathbf{0}$, yields $\mathbb{1}^TW^{-1}\overline{V} = \mathbb{1}^TW^{-1}V^\star$ (Objective 2).

[6] Equations (10c) and (11c) necessarily prescribe that $\alpha(t) = (\mathbb{1}^T(G_l + G_a)\xi_3 - \mathbb{1}^TG_aV(t))/\mathbb{1}^TW^{-1}\mathbb{1}$.

Remark 2. (**Distributed control**). Note that the proposed control strategy is distributed as it prescribes the exchange of information over a communication network on I_t and θ among *only* neighbouring nodes. This implies that each local controller does not require informations on *all* the nodes of the network (see also Assumption 2). As a consequence, the proposed solution is expected to scale well.

Remark 3. (**Robustness**). Although we have provisionally assumed to perfectly know all the network parameters and the *steady state* solution $(\overline{I}_t, \overline{I}, \overline{V}, \overline{\theta})$ to system (4), (9), achieving Objective 1 and Objective 2, the controller output (13) depends on the nominal network voltage $V^\star$, the *actual* value of θ, and the first time derivative of the PCC voltage V_i. The only parameter that is required to be known is the filter inductance L_{ti} (see Assumption 2), while G_a is a design parameter (a tuning rule is provided in Remark 4).

5. STABILITY ANALYSIS

In this section we show that all solutions to (14) converge to a steady state, achieving current sharing (Objective 1) and average voltage regulation (Objective 2). Before doing this, the following assumption is introduced on the existence of a steady state solution to the closed-loop system (14).

Assumption 4. (**Existence of a steady state solution**). There exists a steady state solution $(\overline{I}_t, \overline{I}, \overline{V}, \overline{\theta})$ to system (14), achieving Objectives 1 and 2.

The main result of this work can now be obtained.

Theorem 1. (**Main result**). Let Assumptions 1–4 hold. Consider system (14). The solutions to (14) converge exponentially to a steady state $(\overline{I}_t, \overline{I}, \overline{V}, \overline{\theta}')$, achieving current sharing (Objective 1) and average voltage regulation (Objective 2).

Proof. Let $e := x - \xi$ define the error between the state of system (4), (9) and the corresponding copy (10). Then, the error dynamics are given by:

$$L_t\dot{e}_1 = -e_3 - W\mathcal{L}^ce_4 \tag{15a}$$
$$L\dot{e}_2 = -Re_2 - \mathcal{B}^Te_3 \tag{15b}$$
$$C_t\dot{e}_3 = e_1 + \mathcal{B}e_2 - (G_l + G_a)e_3 \tag{15c}$$
$$\dot{e}_4 = \mathcal{L}^cWe_1. \tag{15d}$$

Consider the incremental storage function

$$\mathcal{S} = \frac{1}{2}e_1^TL_te_1 + \frac{1}{2}e_2^TLe_2 + \frac{1}{2}e_3^TC_te_3 + \frac{1}{2}e_4^Te_4. \tag{16}$$

It is immediate to see that $\mathcal{S}$ is radially unbounded and that $\mathcal{S}$ attains a minimum at $e = \mathbf{0}$. Furthermore, a straightforward calculation shows that $\mathcal{S}$ satisfies

$$\dot{\mathcal{S}} = -e_2^TRe_2 - e_3^T(G_l + G_a)e_3 \leq 0, \tag{17}$$

along the solutions to (15). As an intermediate result, we can conclude that all solutions to system (15) are bounded. According to LaSalle's invariance principle, the solutions to (15) approach the largest invariant set contained entirely in the set

$$\Upsilon = \{e : e_2 = e_3 = \mathbf{0}\}, \tag{18}$$

such that on this set $V = \xi_3 := \overline{V}$ and $I = \xi_2 := \overline{I}$. Furthermore, from (15c) it follows that on this set $e_1 = \mathbf{0}$, i.e., $I_t(t) = \xi_1(t) = W^{-1}\mathbb{1}\alpha(t)$, and, as a consequence, $\dot{e}_4 = \dot{\theta} = \mathbf{0}$, i.e., $\theta := \overline{\theta}'$ is constant and possibly different from $\overline{\theta}$. More precisely, from (14a) at steady state, using (11c), we obtain $\mathcal{L}^c\overline{\theta} = \mathcal{L}^c\overline{\theta}'$, implying that θ converges to $\overline{\theta}' = \overline{\theta} + \mathbb{1}\sigma$, with σ any constant. Finally, we prove that in the set Υ, I_t converges to a constant vector as well. To do this, we observe that in the largest invariant set Υ, $\alpha(t) = \overline{\alpha} = i_t^*$ is constant, with i_t^* defined in Objective 1. Consequently, ξ_1 is a constant vector achieving proportional current sharing (Objective 1), i.e., $I_t = \xi_1 = W^{-1}\mathbb{1}i_t^* := \overline{I}_t$. After pre-multiplying both sides of (14a) with $\mathbb{1}^T W^{-1}$, taking therein $\dot{I}_t = \dot{V} = \mathbf{0}$ and noting that $\mathbb{1}^T\mathcal{L}^c = \mathbf{0}$, it straightforwardly follows that average voltage regulation is guaranteed (Objective 2).

Remark 4. (**Tuning rule for G_a**). Observing that system (4) can be expressed as a Brayton-Moser model (see Brayton and Moser (1964)), a qualitative Lyapunov-based stability condition for system (15) is given by

$$\left|\left|C_t^{\frac{1}{2}} G_d^{-1} \Psi^T \hat{L}^{-\frac{1}{2}}\right|\right| \leq 1 - \beta, \tag{19}$$

with $0 < \beta < 1$, $G_d = G_l + G_a$, $\Psi = [\mathbb{I}, \mathcal{B}^T]^T \in \mathbb{R}^{(n+m)\times n}$, $\mathbb{I} \in \mathbb{R}^{n\times n}$ being the identity matrix, and $\hat{L} = \text{diag}(L_t, L) \in \mathbb{R}^{(n+m)\times(n+m)}$. Fulfilling condition (19) ensures a *nice* response in terms of, e.g., overshoot and settling-time, and can therefore be used to tune the controller suggested in this work [7]. For details we refer the reader to Brayton and Moser (1964).

Remark 5. (**Plug-and-Play**). The main results in this work assume a constant network topology and the analysis of plugging in or out various converters is outside the scope of this work. However, since the convergence result of Theorem 1 holds globally, independently of the initial conditions of the physical power network and the controller state, the proposed solution is expected to be suitable for Plug-and-Play operation.

6. SIMULATION RESULTS

In this section, the proposed passivity-based controller is assessed in simulation. We consider a network composed of four DGUs interconnected as shown in Figure 2, where also the communication network is represented. The parameters of each DGU and the line parameters are reported in Tables 2 and 3, respectively. The weights associated with the edges of the communication graph are $\gamma_{12} = \gamma_{23} = \gamma_{34} = 1 \times 10^2$. In the controller (13), we have selected $G_a = 25 \times \mathbb{I}_4$, $\mathbb{I}_4 \in \mathbb{R}^{4\times 4}$ being the identity matrix.

The system is initially at a steady state with load $G_l(0)$. Then, consider a load variation ΔG_l at the time instant $t = 3\,\text{s}$ (see Table 2). The PCC voltages and the average voltage of the network are illustrated in Figure 3. One can appreciate that the steady state weighted average of the

[7] Note that the tuning rule is derived considering the control problem of regulating the voltage V towards the constant value $V^\star - W\mathcal{L}^c\overline{\theta}$. This is equivalent to assume that the *consensus* dynamic (9) is much faster than the dynamics of the *physical* system (4).

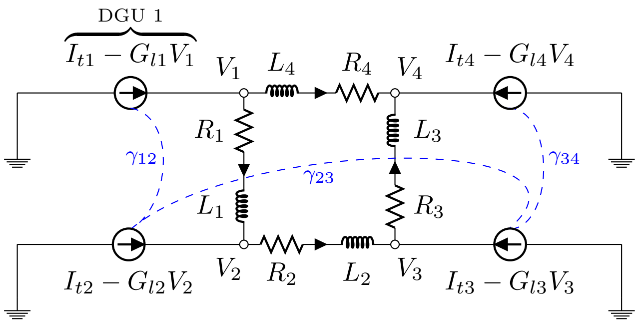

Fig. 2. Scheme of the considered (Kron reduced) network with 4 power converters. The dashed lines represent the communication network.

Table 2. DC Network Parameters

DGU		1	2	3	4
L_{ti}	(mH)	1.8	2.0	3.0	2.2
C_{ti}	(mF)	2.2	1.9	2.5	1.7
w_i	–	0.4^{-1}	0.2^{-1}	0.15^{-1}	0.25^{-1}
$V_i^\star$	(V)	380.0	380.0	380.0	380.0
$G_l(0)$	(Ω^{-1})	0.08	0.04	0.08	0.07
ΔG_l	(Ω^{-1})	0.03	0.02	−0.03	0.01

Table 3. Line Parameters

Line		1	2	3	4
R_k	(mΩ)	70	50	80	60
L_k	(μH)	2.1	2.3	2.0	1.8

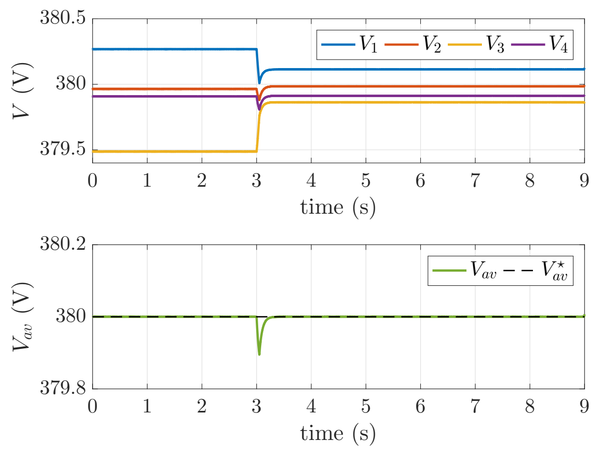

Fig. 3. Time evolution of the voltage at the PCC of each DGU; weighted average value of the network voltages.

PCC voltages (denoted by V_{av}) is equal to the weighted average of the corresponding references (see Objective 2). Figure 4 shows that the current generated by each DGU converges to the desired value, achieving proportional current sharing (see Objective 1). Even if IEEE Standards or guidelines for DC power distribution networks do not exist yet (to the best of our knowledge), it is usually required in practical cases that the voltage deviations are within 5 % of the nominal value. For the presented case, the voltage at the PCC of each DGU is within the range of 380 ± 0.5 V, implying that the voltage deviations are less than 0.15 % of the nominal value $V^\star = 380\,\text{V}$.

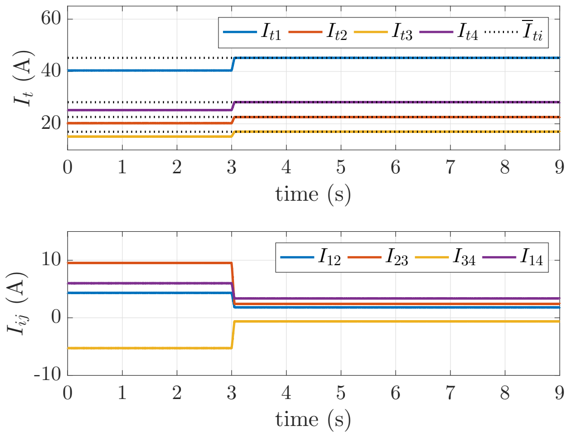

Fig. 4. Time evolution of the generated currents together with the corresponding values (dashed lines) that correspond to (proportional) current sharing for $t > 3$ s; currents shared among DGUs.

7. CONCLUSIONS

In this paper, a distributed passivity-based control scheme is proposed for proportional (fair) current sharing and average voltage regulation in DC power networks, with an arbitrary meshed topology that incorporates dynamic resistive-inductive lines. The control objectives are achieved through a consensus-like algorithm that exploits a communication network, and by injecting damping to modify the dissipation structure of the power network. The controlled network is proven to converge globally to a desired steady state, independently of the initial conditions of the physical power network and the controller state, facilitating scalability and Plug-and-Play capabilities.

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