

Sliding Modes for Voltage Regulation and Current Sharing in DC Microgrids

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Abstract—In this paper a novel distributed control algorithm for voltage regulation and current sharing in Direct Current (DC) microgrids is proposed. The DC microgrid is composed of several Distributed Generation units, interfaced with Buck converters, and unknown current loads. The proposed control strategy exploits a communication network to achieve current sharing using a consensus-like algorithm. Voltage regulation is achieved by constraining the system to a suitable manifold. A third order Sliding Mode controller is developed to reach the desired manifold in a finite time. The proposed control scheme is formally analyzed, proving the achievement of current sharing, while guaranteeing that the average voltage of the microgrid is identical to the average of the voltage references.

I. INTRODUCTION

In the last decades, due to economic, technological and environmental aspects, the main developments in power systems focus are focussed on the large scale deployment of Distributed Generation units (DGUs). Moreover, the ever-increasing energy demand and the concern about the climate change have encouraged the wide diffusion of Renewable Energy Sources (RES). The so-called microgrids have been proposed as conceptual solutions to integrate different types of RES and to electrify remote areas.

Due to the widespread use of Alternate Current (AC) electricity in most industrial, commercial and residential applications, the recent literature on this topic mainly focused on AC microgrids [1]–[4]. However, several sources and loads (e.g. photovoltaic panels, batteries, electronic appliances and electric vehicles) can be directly connected to DC microgrids by using DC-DC converters. Indeed, several aspects make DC microgrids more efficient and reliable than AC microgrids [5]: *i*) lossy DC-AC and AC-DC conversion stages are reduced, *ii*) there is no reactive power, *iii*) harmonics are not present, *iv*) frequency synchronization is overcome, *v*) the skin effect is absent. For all these reasons, DC microgrids are attracting growing interest and receive much research attention.

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Two common control objectives in DC microgrids are voltage regulation and current sharing (or, equivalently, load sharing). Regulating the voltages is required to ensure a proper functioning of connected loads [6]–[8], whereas current sharing prevents overstressing the sources. In order to achieve both objectives, hierarchical control schemes are conventionally adopted [9], [10]. In the literature, these control problems in DC microgrids have been addressed by different approaches (see for instance [11]–[15] and the references there in).

A. Main contributions

This paper proposes a novel robust control algorithm to obtain simultaneously current sharing among the DGUs and a form of voltage regulation in the network. In order to achieve current sharing, a communication network is exploited where each DGU communicates in real-time the value of its generated current to its neighbouring DGUs. In comparison to the existing results in the literature, we additionally propose the design of a manifold that couples the aforementioned objective of current sharing to the objective of voltage regulation. By doing this, the proposed control algorithm guarantees that the average voltage of the microgrid is equal to the average of the reference voltages, which is commonly called voltage balancing [14]. This is achieved independently of the initial voltage conditions, facilitating plug-and-play capabilities. To constrain the state of the system to the designed manifold in a finite time, we rely on Sliding Mode (SM) control methodology [16]. SM control is appreciated for its robustness property against a wide class of modelling uncertainties and external disturbances, commonly present in DC microgrids. In this paper, we propose a third order sliding mode controller (3SM) to obtain a continuous control signal that can be used as the duty cycle of the power converter, achieving constant switching frequency and facilitating the implementation of the Pulse Width Modulation (PWM) technique. For the considered microgrid model, convergence to the state of current sharing and voltage regulation is theoretically analyzed, and we show that convergence is achieved globally, for any initialization of the microgrid.

II. DC MICROGRID MODEL

In this work we consider a typical buck converter-based DC microgrid of which a schematic electrical diagram is provided in Figure 1. By applying the Kirchhoff's current (KCL) and voltage (KVL) laws, the governing dynamic

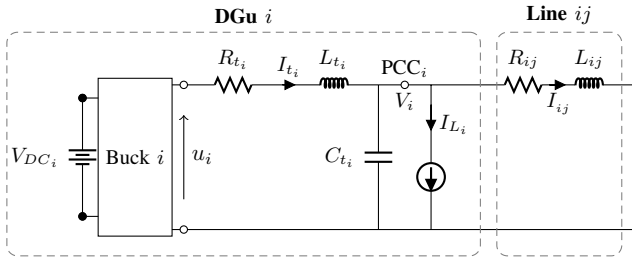


Fig. 1. The considered DC microgrid electrical scheme.

TABLE I
DESCRIPTION OF THE USED SYMBOLS

I_{ti}	Generated current
V_i	Load voltage
I_{ij}	Exchanged current
R_{ti}	Filter resistance
L_{ti}	Filter inductance
C_{ti}	Shunt capacitor
R_i	Line resistance
u_i	Control input
I_{Li}	Unknown current demand

equations* of the i -th node (DGU) are the following:

$$\begin{aligned} L_{ti} \dot{I}_{ti} &= -R_{ti} I_{ti} - V_i + u_i \\ C_{ti} \dot{V}_i &= I_{ti} - I_{Li} - \sum_{j \in \mathcal{N}_i} I_{ij}, \end{aligned} \quad (1)$$

where \mathcal{N}_i is the set of nodes (i.e., the DGUs) connected to the i -th DGU by distribution lines, while the control input u_i represents the controllable buck converter output voltage. The current from DGU i to DGU j is denoted by I_{ij} . Exploiting the Quasi Stationary Line (QSL) approximation of power lines [17], for each $j \in \mathcal{N}_i$, one has[‡]

$$I_{ij} = \frac{1}{R_{ij}} (V_i - V_j). \quad (2)$$

The symbols used in (1) and (2) are described in Table I. The overall network is represented by a connected and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the nodes, $\mathcal{V} = \{1, \dots, n\}$, represent the DGUs and the edges, $\mathcal{E} = \{1, \dots, m\}$, represent the distribution lines interconnecting the DGUs. The network topology is represented by its corresponding incidence matrix $\mathcal{B} \in \mathbb{R}^{n \times m}$. The ends of edge k are arbitrarily labeled with a + and a -, and the entries of \mathcal{B} are given by

$$\mathcal{B}_{ik} = \begin{cases} +1 & \text{if } i \text{ is the positive end of } k \\ -1 & \text{if } i \text{ is the negative end of } k \\ 0 & \text{otherwise.} \end{cases}$$

By substituting (2) in (1), the overall microgrid system can be written compactly for all nodes $i \in \mathcal{V}$ as

$$\begin{aligned} L_t \dot{I}_t &= -R_t I_t - V + u \\ C_t \dot{V} &= I_t - \mathcal{B} R^{-1} \mathcal{B}^T V - I_L, \end{aligned} \quad (3)$$

*For notational simplicity, the dependence of the variables on time t is omitted throughout most of the paper.

where $V, I_t, I_L, u \in \mathbb{R}^n$. Moreover, $C_t, L_t, R_t \in \mathbb{R}^{n \times n}$ and $R \in \mathbb{R}^{m \times m}$ are positive definite diagonal matrices. To permit the controller design in the next sections, the following assumption is introduced on the available information of the system:

Assumption 1: (Available information) The state variables I_{ti} and V_i are locally available at the i -th DGU. The network parameters R_t, L_t, C_t, R , and the current demand I_L are constant and unknown, but with known bounds.

Remark 1: (Kron reduction) Note that in (1), the load currents are located at the PCC of each DGU. This situation is generally obtained by a Kron reduction of the original network, yielding an equivalent representation of the network [12].

III. CURRENT SHARING AND VOLTAGE BALANCING

In this section we make the considered control objectives explicit. First, we note that for a given constant control input \bar{u} , a steady state solution (\bar{I}_t, \bar{V}) to system (3) satisfies[†]

$$\begin{aligned} \bar{V} &= -R_t \bar{I}_t + \bar{u} \\ 0 &= \mathbb{1}_n^T (\bar{I}_t - I_L), \end{aligned} \quad (4)$$

where $\mathbb{1}_n \in \mathbb{R}^n$ is the vector consisting of all ones. The second equation of (4) implies that, at the steady state, the total generated current is equal to the total current demand. To improve the generation efficiency, it is generally desired that the total current demand is shared among the various DGUs (current sharing). This leads us to the first objective concerning the desired steady state value of the generated currents \bar{I}_t [‡].

Objective 1: (Current sharing)

$$\lim_{t \rightarrow \infty} I_t(t) = \bar{I}_t = \mathbb{1}_n i_t^*, \quad (5)$$

with $i_t^* = \frac{1}{n} \mathbb{1}_n^T I_L \in \mathbb{R}$.

From (3) it then follows that the corresponding steady state voltages \bar{V} satisfy $\mathcal{B} R^{-1} \mathcal{B}^T \bar{V} = \mathbb{1}_n i_t^* - I_L$, that prescribes the value of the required differences in voltages, $\mathcal{B}^T \bar{V}$, achieving current sharing. This admits the freedom to shift all steady state voltages with the same constant value, since $\mathcal{B}^T \bar{V} = \mathcal{B}^T (\bar{V} + a \mathbb{1}_n)$, with $a \in \mathbb{R}$ any scalar. To define the optimal steady state voltages, we assume that for every DGU, there exists a desired reference voltage V_i^* .

Assumption 2: (Desired voltages) There exists a constant reference voltage V_i^* at the PCC, for all $i \in \mathcal{V}$.

Generally, the requirement of current sharing does not permit for $\bar{V} = V^*$, and might cause voltages deviations from the corresponding reference values. Then, a reasonable alternative is to keep the average value of the PCC voltages at the steady state identical to the average value of the desired reference voltages of V^* (voltage balancing) [14]. Therefore, given a V^* , we aim at designing a controller that, in addition to Objective 1, also guarantees voltage balancing, i.e.,

[†]The incidence matrix \mathcal{B} satisfies $\mathbb{1}_n^T \mathcal{B} = \mathbf{0}$.

[‡]We refer to the extended version [18] where dynamic (inductive) lines and proportional current sharing are considered.

Objective 2: (Voltage balancing)

$$\lim_{t \rightarrow \infty} \frac{1}{n} \mathbf{1}_n^T V(t) = \frac{1}{n} \mathbf{1}_n^T \bar{V} = \frac{1}{n} \mathbf{1}_n^T V^*. \quad (6)$$

IV. THE PROPOSED SOLUTION: A MANIFOLD-BASED CONSENSUS ALGORITHM

In this section we introduce the key aspects of the proposed solution to achieve Objective 1 and Objective 2. First, we augment system (3) with additional state variables (distributed integrators) θ_i , $i \in \mathcal{V}$, with dynamics given by

$$\dot{\theta}_i = - \sum_{j \in \mathcal{N}_i^c} \gamma_{ij} (I_{t_i} - I_{t_j}), \quad (7)$$

where \mathcal{N}_i^c is the set of the DGUs that communicate with the i -th DGu, and $\gamma_{ij} = \gamma_{ji} \in \mathbb{R}_{\geq 0}$ are additional gain constants. Let \mathcal{L}_c denote the (weighted) Laplacian matrix associated with the connected communication graph, which can be different from the topology of the (reduced) microgrid. Then, the dynamics in (7) can be expressed compactly for all nodes $i \in \mathcal{V}$ as

$$\dot{\theta} = -\mathcal{L}_c I_t, \quad (8)$$

that indeed has the form of a consensus protocol, permitting a steady state where $\bar{I}_t \in \text{im}(\mathbf{1}_n)$ (see also Objective 1). We impose the following restrictions on (8):

Assumption 3: (Controller structure) For all $i \in \mathcal{V}$, the integrators states θ_i are initialized to zero, i.e., $\theta(0) = \mathbf{0}$. Furthermore, the graph corresponding to the topology of the communication network is undirected and connected.

Whereas connectedness of the communication graph is needed to ensure power sharing among *all* DGUs, the consequence of the required initialization of $\theta(0)$ is that the average value of the entries of θ is preserved and identical to zero for all $t \geq 0$, as proved in the following lemma:

Lemma 1: (Preservation of $\mathbf{1}_n^T \theta$) Let Assumption 3 hold. Given system (8), the average value $\frac{1}{n} \sum_{i \in \mathcal{V}} \theta_i$ is preserved, i.e.,

$$\frac{1}{n} \mathbf{1}_n^T \theta(t) = \frac{1}{n} \mathbf{1}_n^T \theta(0) \quad \text{for all } t \geq 0. \quad (9)$$

Proof: Pre-multiplying both sides of (8) by $\mathbf{1}_n^T$ yields

$$\mathbf{1}_n^T \dot{\theta} = -\mathbf{1}_n^T \mathcal{L}_c I_t = 0, \quad (10)$$

where $\mathbf{1}_n^T \mathcal{L}_c = \mathbf{0}$, follows from \mathcal{L}_c being the Laplacian matrix associated with an undirected graph. ■

The fact that $\mathbf{1}_n^T \theta(t) = 0$, is essential to the second aspect of the proposed solution, the design of a manifold. Bearing in mind Objective 2, aiming at voltage balancing where

$$\lim_{t \rightarrow \infty} \frac{1}{n} \mathbf{1}_n^T V(t) = \frac{1}{n} \mathbf{1}_n^T \bar{V} = \frac{1}{n} \mathbf{1}_n^T V^*, \quad (11)$$

we propose the following desired manifold:

$$\{(I_t, V, \theta) : V - V^* - \theta = \mathbf{0}\}. \quad (12)$$

Indeed, exploiting the preservation of $\mathbf{1}_n^T \theta$, we have on the desired manifold (12), $\mathbf{1}_n^T V = \mathbf{1}_n^T (\theta + V^*) = \mathbf{1}_n^T V^*$. Constraining the solutions to a system to a specific manifold is typical for sliding mode based controllers, and we will discuss some suitable controller in the next section.

V. SLIDING MODE CONTROLLERS

We now propose a Distributed Third Order Sliding Mode (D-3SM) control law, to steer, in a finite time, the state of system (3), augmented with (8), to the desired manifold (12). Bearing in mind the desired manifold (12), we consider the following sliding function $\sigma \in \mathbb{R}^n$:

$$\sigma(V, \theta) = V - V^* - \theta. \quad (13)$$

Regarding the sliding function (13) as the output function of system (3), (8), it appears that the relative degree is two. This implies that a second order sliding mode (SOSM) controller can be *naturally* applied in order to make the state of the controlled system reach, in a finite time, the sliding manifold. As is typical for sliding mode controllers, the resulting control input would be discontinuous. However, in order to achieve a constant switching frequency, Buck converters are controlled by implementing the so-called Pulse Width Modulation (PWM) technique. To do this, a continuous control signal, that represents the so-called duty cycle of the Buck converter, is required. To ensure a continuous control input (duty cycle), we adopt the procedure suggested in [19] and first integrate the (discontinuous) control signal generated by a sliding mode controller, yielding for system (3) augmented with (8)

$$\begin{aligned} L_t \dot{I}_t &= -R_t I_t - V + u \\ C_t \dot{V} &= I_t - \mathcal{B} R^{-1} \mathcal{B}^T V - I_L \\ \dot{\theta} &= -\mathcal{L}_c I_t \\ \dot{u} &= v, \end{aligned} \quad (14)$$

where v is the new (discontinuous) control input. Note that the input signal to the converter, $u(t) = \int_0^t v(\tau) d\tau$, is continuous, so that u_i can be used as duty cycle for the switch of the i -th Buck converter. A consequence is that the system relative degree (with respect to the new control input v) is now equal to three, so that we need to rely on a third order sliding mode (3SM) control strategy. To do so, we define the auxiliary variables $\xi_1 = \sigma$, $\xi_2 = \dot{\sigma}$ and $\xi_3 = \ddot{\sigma}$, and build the auxiliary system as follows

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \xi_3 \\ \dot{\xi}_3 &= \beta(\dot{I}_t, \dot{V}, \dot{I}_L, v) + G_d v \\ \dot{u} &= v, \end{aligned} \quad (15)$$

where $\beta \in \mathbb{R}^n$, is given by

$$\begin{aligned} \beta &= -((C_t^{-1} + \mathcal{L}_c) L_t^{-1} R_t + C_t^{-1} \mathcal{B} R^{-1} \mathcal{B}^T C_t^{-1}) \dot{I}_t \\ &\quad + (C_t^{-1} \mathcal{B} R^{-1} \mathcal{B}^T C_t^{-1} \mathcal{B} R^{-1} \mathcal{B}^T - (C_t^{-1} + \mathcal{L}_c) L_t^{-1}) \dot{V} \\ &\quad + C_t^{-1} \mathcal{B} R^{-1} \mathcal{B}^T C_t^{-1} \dot{I}_L - G_a v, \end{aligned} \quad (16)$$

with G_d and G_a given by

$$\begin{aligned} G_d &= (C_t^{-1} + \mathcal{D}_c) L_t^{-1}, \\ G_a &= \mathcal{A}_c L_t^{-1}. \end{aligned} \quad (17)$$

Here, \mathcal{D}_c and \mathcal{A}_c are the degree matrix and the adjacency matrix of the communication graph, respectively, i.e. $\mathcal{L}_c =$

$\mathcal{D}_c - \mathcal{A}_c$. Then, we assume that for any $i \in \mathcal{V}$, the entries of β and G_d can be bounded as

$$\begin{aligned} |\beta_i(\cdot)| &\leq \beta_{\max_i} \\ G_{\min_i} &\leq G_{d_i} \leq G_{\max_i}, \end{aligned} \quad (18)$$

where β_{\max_i} , G_{\min_i} and G_{\max_i} are known positive constants.

To make the controller design explicit, we consider the 3SM control law proposed in [20] to steer ξ_{1_i} , ξ_{2_i} and ξ_{3_i} , $i \in \mathcal{V}$, to zero in a finite time, i.e.,

$$v_i = -\alpha_i \begin{cases} v_{1_i} = \text{sgn}(\ddot{\sigma}_i) & \sigma_i \in \mathcal{M}_{1_i}/\mathcal{M}_{0_i} \\ v_{2_i} = \text{sgn}\left(\dot{\sigma}_i + \frac{\ddot{\sigma}_i v_{1_i}}{2\alpha_{r_i}}\right) & \sigma_i \in \mathcal{M}_{2_i}/\mathcal{M}_{1_i} \\ v_{3_i} = \text{sgn}(s_i(\sigma_i)) & \text{otherwise,} \end{cases} \quad (19)$$

where $\sigma_i = [\sigma_i, \dot{\sigma}_i, \ddot{\sigma}_i]^T$ and

$$s_i(\sigma_i) = \sigma_i + \frac{\ddot{\sigma}_i^3}{3\alpha_{r_i}^2} + v_{2_i} \left[\frac{1}{\sqrt{\alpha_{r_i}}} \left(v_{2_i} \dot{\sigma}_i + \frac{\ddot{\sigma}_i^2}{2\alpha_{r_i}} \right)^{\frac{3}{2}} + \frac{\dot{\sigma}_i \ddot{\sigma}_i}{\alpha_{r_i}} \right],$$

with $\alpha_{r_i} = \alpha_i G_{\min_i} - \beta_{\max_i} > 0$. Then, given the bounds G_{\min_i} and β_{\max_i} , the control amplitude α_i is chosen such that α_{r_i} is positive. The manifolds \mathcal{M}_{1_i} , \mathcal{M}_{2_i} , \mathcal{M}_{3_i} in (19) are defined as

$$\begin{aligned} \mathcal{M}_{0_i} &= \{ \sigma_i \in \mathbb{R}^3 : \sigma_i = \dot{\sigma}_i = \ddot{\sigma}_i = 0 \} \\ \mathcal{M}_{1_i} &= \{ \sigma_i \in \mathbb{R}^3 : \sigma_i - \frac{\ddot{\sigma}_i^3}{6\alpha_{r_i}^2} = 0, \dot{\sigma}_i + \frac{\ddot{\sigma}_i |\dot{\sigma}_i|}{2\alpha_{r_i}} = 0 \} \\ \mathcal{M}_{2_i} &= \{ \sigma_i \in \mathbb{R}^3 : s_i(\sigma_i) = 0 \}. \end{aligned}$$

From (19), one can observe that the controller of DGU i requires not only σ_i , but also $\dot{\sigma}_i$ and $\ddot{\sigma}_i$. Yet, according to Assumption 1, only I_t and V_i are measurable at the i -th DGU. Then, one can rely on Levant's second-order differentiator [21] to retrieve $\dot{\sigma}_i$ and $\ddot{\sigma}_i$ in a finite time.

Remark 2: (Scalability and distributed control) Since the sliding function (13) is designed by using the additional state θ in (8), the overall control scheme is distributed. More precisely, the controller of the DGU i needs information on the generated currents I_t only from the DGUs that communicate with it. Note that the design of the local controller for each DGU is not based on the knowledge of the whole microgrid, so that the complexity of the control synthesis does not depend on the microgrid size.

VI. STABILITY ANALYSIS

In this section we first show that the states of the controlled microgrid are constrained, after a finite time, to the manifold $\sigma = \mathbf{0}$, where Objective 2 is achieved. Thereafter, we prove that the solutions to the system, once the sliding manifold is attained, converge exponentially to a constant point, achieving additionally Objective 1. As a first step, we study the convergence to the sliding manifold when the 3SM control law is applied to the system.

Lemma 2: (Convergence to the sliding manifold: 3SM) Let Assumption 1 hold. The solutions to system (3), (8), controlled via the 3SM control algorithm (15), converge in a finite time T_r , to the sliding manifold $\{(I_t, V, \theta) : \sigma = \dot{\sigma} = \ddot{\sigma} = \mathbf{0}\}$, with σ given by (13).

Proof: The details are omitted, since they are an immediate consequence of the used 3SM algorithm [20]. ■

Relying on the proposed sliding function (13), the *equivalent control*[§] v_{eq} can be derived from (15) by posing $\sigma^{(3)} = \dot{\xi}_3 = \mathbf{0}$. More precisely, define $\tilde{\beta} = \beta + G_d v$, then v_{eq} is given by

$$v_{eq} = -(G_d - G_a)^{-1} \tilde{\beta} \quad \forall t \geq T_r. \quad (20)$$

Since $v = \dot{u}$, then the equivalent version of the control input actually fed into the plant is computed by integrating (20). Therefore, for any $t \geq T_r$, $u_{eq}(t) = \int_{T_r}^t v(\tau) d\tau$ is given by

$$\begin{aligned} u_{eq} &= (R_t + L_t(C_t^{-1} + \mathcal{L}_c)^{-1} C_t^{-1} \mathcal{B} R^{-1} \mathcal{B}^T C_t^{-1}) I_t \\ &\quad + (I_{n \times n} - L_t(C_t^{-1} + \mathcal{L}_c)^{-1} C_t^{-1} \\ &\quad \cdot \mathcal{B} R^{-1} \mathcal{B}^T C_t^{-1} \mathcal{B} R^{-1} \mathcal{B}^T) V \\ &\quad - L_t(C_t^{-1} + \mathcal{L}_c)^{-1} C_t^{-1} \mathcal{B} R^{-1} \mathcal{B}^T C_t^{-1} I_L. \end{aligned} \quad (21)$$

Once the sliding manifold is attained, the dynamics of system (3), (8), are described by the so-called *equivalent system* obtained by substituting u_{eq} for u .

Lemma 3: (Equivalent system) For all $t \geq T_r$, the dynamics of the controlled microgrid are given by the equivalent version of system (3), (8), i.e.,

$$\begin{aligned} \dot{I}_t &= -A I_t \\ \dot{\theta} &= -\mathcal{L}_c I_t, \end{aligned} \quad (22)$$

with

$$A = (I_{n \times n} + C_t \mathcal{L}_c)^{-1} \mathcal{B} R^{-1} \mathcal{B}^T \mathcal{L}_c. \quad (23)$$

Proof: After substituting expression (21) for u in (3), the dynamics of the generated current I_t become

$$\begin{aligned} \dot{I}_t &= (I_{n \times n} + C_t \mathcal{L}_c)^{-1} \mathcal{B} R^{-1} \mathcal{B}^T C_t^{-1} I_t \\ &\quad - (I_{n \times n} + C_t \mathcal{L}_c)^{-1} \mathcal{B} R^{-1} \mathcal{B}^T C_t^{-1} \mathcal{B} R^{-1} \mathcal{B}^T V \\ &\quad - (I_{n \times n} + C_t \mathcal{L}_c)^{-1} \mathcal{B} R^{-1} \mathcal{B}^T C_t^{-1} I_L. \end{aligned} \quad (24)$$

Moreover, the sliding constraint $\dot{\sigma} = \mathbf{0}$, implies that $\dot{V} = \dot{\theta}$ for all $t \geq T_r$. Then, one can straightforwardly obtain the following algebraic relation:

$$\mathcal{B} R^{-1} \mathcal{B}^T V = (I_{n \times n} + C_t \mathcal{L}_c) I_t - I_L. \quad (25)$$

Finally, (22) is obtained by substituting (25) in (24). ■

Before studying the stability of the equivalent system, we prove a useful result in the lemma below.

Lemma 4: ($(P - (P^{-1} + Q)^{-1}) \succeq 0$) Given a positive definite matrix $P \in \mathbb{R}^{n \times n}$ and a positive semidefinite matrix $Q \in \mathbb{R}^{n \times n}$, then $P - (P^{-1} + Q)^{-1} \succeq 0$.

Proof: Let $\tilde{Q} = P^{\frac{1}{2}} Q P^{\frac{1}{2}}$. Clearly, $\tilde{Q} \succeq 0$, and $I_{n \times n} + \tilde{Q} \succ 0$. Then, $P - (P^{-1} + Q)^{-1} = P^{\frac{1}{2}} \left[I_{n \times n} - (I_{n \times n} + \tilde{Q})^{-1} \right] P^{\frac{1}{2}}$ is a positive semidefinite matrix if and only if $I_{n \times n} - (I_{n \times n} + \tilde{Q})^{-1} = \tilde{Q} (I_{n \times n} + \tilde{Q})^{-1} \succeq 0$. Observing that $(I_{n \times n} + \tilde{Q})^{-1} \succ 0$, it yields

[§]The *equivalent control* describes the *average* effect of the discontinuous control when the controlled system is constrained to the sliding manifold. It is a conventional tool in the analysis of sliding mode control systems [16].

$\tilde{Q}(I_{n \times n} + \tilde{Q})^{-1} \succsim (I_{n \times n} + \tilde{Q})^{-\frac{1}{2}} \tilde{Q} (I_{n \times n} + \tilde{Q})^{-\frac{1}{2}} \succeq 0$, which completes the proof. ■

We can now establish the following properties of matrix \mathcal{A} that are essential to the stability analysis:

Lemma 5: (Properties of \mathcal{A}) Matrix \mathcal{A}

- (i) has nonnegative eigenvalues;
- (ii) has a zero eigenvalue, with algebraic multiplicity one;
- (iii) satisfies $\ker(\mathcal{A}) = \text{im}(\mathbf{1}_n)$.

Proof: Basic algebraic manipulations show that

$$\mathcal{A} = X^{-1} C_t^{-1} B R^{-1} B^T X - X^{-1} C_t^{-1} B R^{-1} B^T C_t^{-1}, \quad (26)$$

with $X = C_t^{-1} + \mathcal{L}_c$ being a positive definite matrix. After the similarity transformation X , preserving the eigenvalues, we have

$$\mathcal{A} \succsim \underbrace{C_t^{-1} B R^{-1} B^T C_t^{-1}}_S \underbrace{(C_t - X^{-1})}_Z = \tilde{\mathcal{A}}. \quad (27)$$

According to Lemma 4 (considering $P = C_t$ and $Q = \mathcal{L}_c$), $Z \succeq 0$. Observing that $S \succeq 0$, then, $\tilde{\mathcal{A}}$ has non-negative eigenvalues as it is a product of two positive semi-definite matrices [22, Corollary 8.3.6] (i). From (23) one can straightforwardly establish that $\mathcal{A}x = \mathbf{0}$ if and only if $x \in \text{im}(\mathbf{1}_n)$ (iii). Moreover, since two positive semi-definite matrices are simultaneously diagonalizable, the algebraic multiplicity of the zero eigenvalue associated to the eigenvector $\mathbf{1}_n$ is identical to its geometric multiplicity, which is one (ii). ■

We can now establish the first main result of this paper.

Theorem 1: (Achieving current sharing) Let Assumptions 1–3 hold. Consider system (3), (8), controlled with the proposed distributed 3SM control scheme. Then, the generated currents $I_t(t)$ converge, after a finite time, exponentially to $\frac{1}{n} \mathbf{1}_n \mathbf{1}_n^T I_L$, achieving current sharing.

Proof: According to Lemma 3, for all $t \geq T_r$, the dynamics of the generated currents I_t are given by the autonomous system

$$\dot{I}_t = -\mathcal{A}I_t, \quad (28)$$

with \mathcal{A} as in (23). Bearing in mind the properties established in Lemma 5, the matrix $-\mathcal{A}$ is semistable [23, Proposition 1] and therefore $\lim_{t \rightarrow \infty} I_t(t)$ exists for all initial conditions $I_t(T_r)$. Since (28) is linear and $\ker(\mathcal{A}) = \text{im}(\mathbf{1}_n)$, the solution to system (28), with initial condition $I_t(T_r)$, converges exponentially to a constant vector, achieving current sharing. ■

Exploiting Theorem 1, we proceed with establishing the second main result of this paper.

Theorem 2: (Achieving voltage balancing) Let Assumptions 1–3 hold. Consider system (3), (8), controlled with the proposed distributed 3SM control scheme. Then, given a desired references vector V^* , the voltages $V(t)$ satisfy $\frac{1}{n} \mathbf{1}_n^T V(t) = \frac{1}{n} \mathbf{1}_n^T V^*$ for all $t \geq T_r$, with T_r a finite time. Furthermore, from time T_r , the voltages $V(t)$ converges exponentially to a constant vector.

Proof: Following Lemma 2, for all $t \geq T_r$, the equality $V(t) = V^* + \theta(t)$ holds. Pre-multiplying both sides by $\mathbf{1}_n^T$

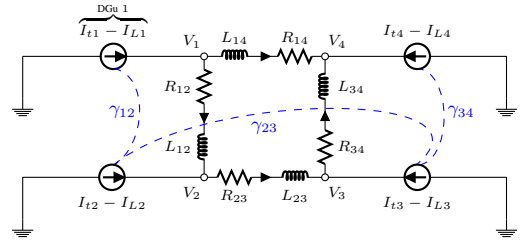


Fig. 2. Scheme of the considered (Kron reduced) microgrid with 4 power converters. The dashed lines represent the communication network.

TABLE II
MICROGRID PARAMETERS AND CURRENT DEMAND

DGu		1	2	3	4
R_{t_i}	(Ω)	0.2	0.3	0.5	0.1
L_{t_i}	(mH)	1.8	2.0	3.0	2.2
C_{t_i}	(mF)	2.2	1.9	2.5	1.7
V_i^*	(V)	380.0	380.0	380.0	380.0
$V_i(0)$	(V)	380.2	380.05	379.95	379.8
$I_{L_i}(0)$	(A)	25.0	15.0	10.0	30.0
ΔI_{L_i}	(A)	5.0	7.5	12.5	-5.0

TABLE III
LINE PARAMETERS

Line		{1,2}	{2,3}	{3,4}	{1,4}
R_{ij}	(m Ω)	70	50	80	60
L_{ij}	(μ H)	2.1	2.3	2.0	1.8

yields $\mathbf{1}_n^T V(t) = \mathbf{1}_n^T V^* + \mathbf{1}_n^T \theta(t)$. Due to Assumption 3 and by virtue of Lemma 1, one has that $\mathbf{1}_n^T \theta(t) = \mathbf{1}_n^T \theta(0) = 0$. Then, one can conclude that voltage balancing is achieved for all $t \geq T_r$. Furthermore, according to Lemma 3, for all $t \geq T_r$, the dynamics of the controlled microgrid are given by the autonomous system (22). We established in Theorem 1, that for system (22), I_t converges exponentially to a constant vector in $\text{im}(\mathbf{1}_n)$. Consequently, the right hand side of (22) vanishes exponentially, such that θ converges exponentially to a constant vector. Therefore, apart from achieving voltage balancing, from $t \geq T_r$, the voltages V converge exponentially to a constant vector as well. ■

Remark 3: (Robustness to failed communication) By omitting the variable θ in the analysis, the controlled microgrid converges, in a finite time, to the manifold $\sigma = \mathbf{0}$, where $V = V^*$, as shown in [7].

VII. SIMULATION RESULTS

In this section, the proposed control scheme is assessed in simulation, considering a microgrid composed of 4 DGUs interconnected as shown in Figure 2, where also the communication network is depicted. The parameters of each DGU and the line parameters are reported in Tables II and III, respectively. The weights associated with the edges of the communication graph are $\gamma_{12} = \gamma_{23} = \gamma_{34} = 1 \times 10^3$. For all the DGUs the controller parameter α_i in (19) is set to 2.5×10^3 . The system is initially at the steady state. Then, consider a current demand variation ΔI_{L_i} at the time instant $t = 0.1$ s. The PCC voltages and the generated currents are illustrated in Figure 3, where the average of the PCC

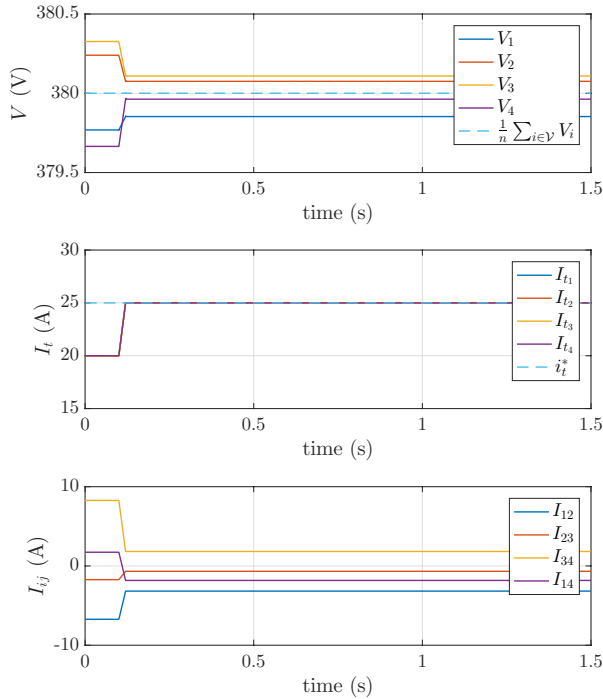


Fig. 3. From the top: voltage at the PCC of each DGU together with its average value (dashed line); generated currents together with the corresponding value (dashed line) that allows to achieve current sharing; currents shared among the DGUs through the lines.

voltages is always equal to the average of the corresponding references, and the current generated by each DGU converges to the desired value $i_t^* = 25$ A, achieving current sharing. At the bottom of Figure 3 the currents shared among the DGUs are also reported.

VIII. CONCLUSIONS

In this paper we have developed a distributed control algorithm, obtaining current sharing and voltage regulation in DC microgrids. Its convergence properties are analytically investigated, and a case study shows the effectiveness of the proposed solution. The proposed control scheme exploits a communication network to achieve current sharing using a consensus-like algorithm. Another useful feature of the proposed control scheme is that the average voltage of the microgrid converges to the average of the voltage references, independently of the initial voltage conditions. The latter is achieved by constraining the system to a suitable manifold. To ensure that the desired manifold is reached in a finite time, even in presence of modelling uncertainties, a third order sliding mode control strategy is proposed, that provides the duty cycle of the power converters.

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