# Highway Traffic Control via Smart e-Mobility - Part I: Theory 

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#### Abstract

In this paper, we study how to alleviate highway traffic congestion by encouraging plug-in hybrid and electric vehicles to stop at a charging station around peak congestion times. Specifically, we design a pricing policy to make the charging price dynamic and dependent on the traffic congestion, predicted via the cell transmission model, and the availability of charging spots. Furthermore, we develop a novel framework to model how this policy affects the drivers' decisions by formulating a mixedinteger potential game. Technically, we introduce the concept of "road-to-station" (r2s) and "station-to-road" (s2r) flows, and show that the selfish actions of the drivers converge to charging schedules that are individually optimal in the sense of Nash. In the second part of this work, submitted as a separate paper (Part II: Case Study), we validate the proposed strategy on a simulated highway stretch between The Hague and Rotterdam, in The Netherlands.


## I. Introduction

## A. Motivation

IN the recent years, urban mobility in highly populated cities is becoming a central issue in many countries. Some alarming statistics show a pressing need for change, as the cost of congestion to the EU society is no less than $€ 267$ billion per year [1]. In fact, an inefficient transportation system deteriorates not only the citizens' well-being, but also the environment, since traffic jams heavily increase the emission of $\mathrm{CO}_{2}$ [2]. The classical solution to the Traffic Demand Management (TDM) problem is to increase the roads' capacity or to build alternative routes. Although this approach produces tangible benefits [3], policymakers and researchers are exploring alternatives that may be sensibly faster and cheaper to implement, and provide dynamic solutions that adapt to the traffic evolution.

## B. "Hard" and "soft" policies

In the past years, there has been a growing interest from the research community in the problem of Active Traffic
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Demand Management (ATDM), i.e., a dynamic or even realtime solution to the traffic control problem. The literature on the topic can be divided into the design of "hard" and "soft" policies to address the problem [4]. The hard type of measures tries to enforce changes in the drivers' behavior by imposing some constraints or penalizing undesired actions. For example, several works studied the use of dynamic traffic signaling or traffic lights to influence the current traffic flow [5], [6]. In [7], the authors impose an artificial bottleneck to decrease the flow in strategic part of the road and achieve an alleviation of the congestion. Another approach is to increase the transit price of the most congested roads in order to boost the use of alternative routes [8].

On the other hand, the so-called soft measures are designed to incentivize virtuous driver behaviors, and have their roots in behavioural economics and psychology. The word soft refers to the possibility of the drivers to ignore the incentives and stick to their regular conduct [4]. Usually, these policies do not imply any physical change of the infrastructure. In fact, they rely on economic incentives or leverage psychological phenomena to change the drivers' habits. Most of the solutions based on this approach lack strong theoretical fundations, and an a posteriori analysis is performed to study their consequences. In [9], the author explores the effectiveness of: monetary incentives, gifts and social nudges tapping into altruistic values. A personalized set of incentives (mostly monetary) is proposed in [10], where a platform is introduced that enables the commuters to receive incentives if they change their departure time to off-peak hours or use an alternative. Several other pilot studies have been performed and they have experimentally validated the benefits of soft policies, see [11] among others. It is important to stress that these two classes of measures are not always mutually exclusive but they can be used in combination to amplify the final effect of congestion alleviation, as we advocate in this paper.

## C. Smart charging of Plug-in Electric Vehicles (PEVs)

The continuous growth of the number of PEVs is also due to the improvements in the smart charging, allowing the vehicles to charge up to 150 kW . This technology increases the appeal of short stops for the users, making the PEVs more similar to fuel vehicles. This motivates several studies on how the PEV drivers may optimize their charging schedules and how they affect the distribution network. Some classic results [12], [13], tackle the problem of high peaks in the energy demand by proposing a dynamic energy price that leads to a change
in the charging habits of the PEV owners, and consequently to the so-called "valley filling" effect [14]. Some recent works considered smart charging coupled with mobility. In [15], the smart charging problem is enhanced by considering also the travel habits. However, the goal is solely to decrease the energy peak demand rather than the traffic congestion level. Other works focus on optimizing the charging of the PEVs to decrease their travel time [16], [17]. However, in these works the overall congestion level is not taken into account in the decision process. For this reason, we cannot consider these solutions as a form of ATDM. To the best of the authors' knowledge, it is still an open, yet appealing, problem to develop ATDM strategies based on smart-charging of PEVs whose main goal is traffic congestion alleviation.

## D. Paper contribution

Inspired by the conventional ramp metering control and motivated by the rising number of PEVs, we propose for the first time a novel ATDM based on soft measures (via monetary incentives) that leverages smart fast charging of PEVs in the road to alleviate traffic congestion during rush hours. Specifically, we propose a dynamic energy price discounted proportionally to the (predicted) congestion level. This approach encourages the PEV owners to stop for charging when the congestion level is (going to be) high, thus aligning the goal of the traffic control with the drivers' self-interest. In the following, we emphasize our main contributions:

- We use for the first time the electricity price as control input for the ATDM. While historically, traffic control had suffered from a lack of control means, this additional control input may prove itself essential to achieve the desired results by acting in synergy with the classical TDM.
- We enrich the Cell Transmission Model (CTM) with the introduction of road-to-station (r2s) and station-to-road (s2r) flows, newly defined to model the entering and leaving of the PEVs in and out of a Charging Station (CS).
- We carry out a formal analysis of the effects of the presented soft policy by describing the decision process of the PEV drivers as a generalized exact potential game.
- We propose a semi-decentralized control scheme ensuring that the PEVs involved in the decision reach an optimal charging schedule that represents their individual best trade-off between monetary saving and travel time.
In the second part of this work [18](Part II: Case Study), we validate this ATDM strategy on a simulated highway stretch between The Hague and Rotterdam, in The Netherlands.


## II. Cell Transmission Model with Charging Station

We consider a freeway stretch without ramps and only one CS where PEVs may stop. In the literature, the most used model for traffic control is the CTM, see [19].

Here, we explicitly introduce a revised version of the CTM described in [20, Sec. 3.3.1] adapted to our problem. We consider the discretized version of the model where each


Fig. 1: Partitioning of the highway in cells and CS for the PEVs; compact graphical representation of the CTM and the notation for the first two cells.
time interval $[k T,(k+1) T)$ of length $T$ is denoted by an integer $k \in \mathbb{N}$. The highway stretch is modeled as a chain of $N$ subsequent cells (Figure 11. The vehicles in each cell $\ell \in \mathcal{N}:=\{1, \ldots, N\}$ are a mixture of PEVs and nonPEVs moving at a constant speed. Two subsequent cells are connected via an interface that models a certain flow of vehicles, whose value depends on the cells' demand and supply capabilities. Without loss of generality, we assume that the CS is located between the first two cells. To formalize the CTM, for every cell $\ell \in \mathcal{N}$ and interval $k \in \mathbb{N}$, we introduce the following set of variables:

- $\rho_{\ell}(k)[\mathrm{veh} / \mathrm{km}]$ : traffic density of cell $\ell$ during $k$;
- $\Phi_{\ell}^{+}(k)[\mathrm{veh} / \mathrm{h}]$ (resp. $\Phi_{\ell}^{-}(k)$ ): total flow entering (exiting) the cell $\ell$ during $k$;
- $\phi_{\ell}(k)[\mathrm{veh} / \mathrm{h}]$ : flow entering cell $\ell$ from cell $\ell-1$ during $k ; \phi_{1}(k)$ (resp. $\phi_{N+1}(k)$ ) is the flow entering (exiting) the highway during the same interval;
We enrich the conventional CTM introducing two flows:
- $\mathrm{r} 2 \mathrm{~s}(k)[\mathrm{veh} / \mathrm{h}]$ : flow of PEVs entering the CS during $k$;
- $\operatorname{s} 2 \mathrm{r}(k)[\mathrm{veh} / \mathrm{h}]$ : flow of PEVs exiting the CS during $k$.

Then, we associate a set of fixed parameter to each cell $\ell$ :

- $L_{\ell}[\mathrm{km}]$ : cell length;
- $\bar{v}_{\ell}[\mathrm{km} / \mathrm{h}]$ : free-flow speed;
- $w_{\ell}[\mathrm{km} / \mathrm{h}]$ : congestion wave speed;
- $q_{\ell}^{\max }[\mathrm{veh} / \mathrm{h}]$ : maximum cell capacity;
- $\rho_{\ell}^{\max }[\mathrm{veh} / \mathrm{km}]:$ maximum jam density.

Each cell can be seen as an input-output system where the inflow is the input and the outflow the output. The dynamics of the density $\rho_{\ell}$ of cell $\ell \in \mathcal{N}$ read as

$$
\begin{equation*}
\rho_{\ell}(k+1)=\rho_{\ell}(k)+\frac{T}{L_{\ell}}\left(\Phi_{\ell}^{+}(k)-\Phi_{\ell}^{-}(k)\right) \tag{1}
\end{equation*}
$$

where the inflow and outflow are defined as

$$
\begin{align*}
\Phi_{\ell}^{-}(k) & := \begin{cases}\phi_{\ell+1}(k)+\mathrm{r} 2 \mathrm{~s}(k) & \text { if } \ell=1 \\
\phi_{\ell+1}(k) & \text { otherwise }\end{cases}  \tag{2a}\\
\Phi_{\ell}^{+}(k) & := \begin{cases}\phi_{\ell}(k)+\mathrm{s} 2 \mathrm{r}(k) & \text { if } \ell=2 \\
\phi_{\ell}(k) & \text { otherwise }\end{cases} \tag{2b}
\end{align*}
$$

Thus, the flows entering and exiting the CS modify only the definition of the in-flow of cell 2 and out-flow from cell 1.

Remark 1 (r2s and s2r flows): The concepts of r2s and s2r flows are inspired by the "off-ramp" and "on-ramp" flows [19], respectively, and used to model the temporary stop of some

PEVs at the CS, which leads, differently from the off- and onramp flows, to a mutual dependency between r2s and s2r. We investigate this further in Sections III and $V$ In the literature, only the on-ramp flow can be controlled, e.g. via a toll, while our control action influences the off-ramp flow as well.

The demand $D_{\ell-1}(k)$ of cell $\ell-1$ and the supply $S_{\ell}(k)$ of cell $\ell$ directly influence the admissible flow between the two cells. The former is the flow that cell $\ell-1$ can send to cell $\ell$ in the time interval $k$, while $S_{\ell}(k)$ describes the flow that cell $\ell$ can receive in the same interval:

$$
\begin{align*}
D_{\ell-1}(k) & :=\min \left\{\bar{v}_{\ell-1} \rho_{\ell-1}(k), q_{\ell-1}^{\max }\right\},  \tag{3a}\\
S_{\ell}(k) & :=\min \left\{w_{\ell}\left(\rho_{\ell}^{\max }-\rho_{\ell}(k)\right), q_{\ell}^{\max }\right\} . \tag{3b}
\end{align*}
$$

The relations in (3) directly define $\phi_{\ell}(k)$ in 2). In fact, if $\ell \in\{3, \ldots, N\}$, then the flow between the cells reads as $\phi_{\ell}:=\min \left\{D_{\ell-1}(k), S_{\ell}(k)\right\}$. On the other hand, the flow $\phi_{2}$ between cell 1 and 2 is described by a more complex relation, due to the presence of the CS:

$$
\begin{cases}\phi_{2}:=D_{1}-\mathrm{r} 2 \mathrm{~s} & \text { if } D_{1}-\mathrm{r} 2 \mathrm{~s} \leq S_{2}-\mathrm{s} 2 \mathrm{r}  \tag{4}\\ \phi_{2}:=S_{2}-\mathrm{s} 2 \mathrm{r} & \text { otherwise }\end{cases}
$$

where the time dependency is omitted. The first case in (4) reflects the free-flow scenario, while the second reflects the presence of a congestion, as the supply of cell 2 is saturated by $\phi_{2}(k)$ and $\mathrm{s} 2 \mathrm{r}(k)$. Finally, $\phi_{1}(k)$ and $\phi_{N+1}(k)$ are the input and output flows of the CTM, respectively.

Throughout this section, we have defined the whole CTM except for $\mathrm{r} 2 \mathrm{~s}(k)$ and $\mathrm{s} 2 \mathrm{r}(k)$. The remainder of the paper is devoted to design the decision process that the PEVs carry out to choose whether or not it is worth stopping at the CS. In turn, this determines $\mathrm{r} 2 \mathrm{~s}(k)$ and $\mathrm{s} 2 \mathrm{r}(k)$, as we show in Section V

## III. DECISION MAKING PROCESS

We assume the presence of a Highway Operator (HO) that aims at minimizing the congestion. It does that by discounting the energy price if the level of congestion grows (or is expected to grow). In this setup, the HO would have the role of the so-called choice architect, by designing the price at all time intervals. Our solution leverages two main aspects: first, if the road is congested, the benefit of keep driving decreases, due to a longer travel time, and, at the same time, stopping at a CS to charge becomes more profitable due to an energy price discount. Secondly, we take advantage of the range anxiety, which is a well-known cognitive bias affecting PEV drivers making them impatient to stop at a CS even if they do not strictly need it [21].

We model the multi-agent decision process of the PEVs exiting cell 1 , at every time interval $k$, by defining a set of interdependent local optimization problems. Each PEV (or agent/player) aims at minimizing its own local cost function subject to a set of constraints, where couplings between the agents arise in both the cost functions and the constraints. From a mathematical point of view, specific in our problem setup, the collection of all these optimization problems determines a mixed-integer potential game subject to bestresponse dynamics. The output of the decision making process
(or game) is the set of all the choices (or strategies) of the PEVs to stop or not at the CS, which affects the r2s flow. The $s 2 r$ flow is instead a consequence of how long the agents decide to linger at the CS.

## A. Cost function

Next, we design the cost function of the PEVs exiting cell 1 that are involved in the decision making process. We postulate that the interest of each driver is twofold: he is interested in minimizing the travel time, while he is also willing to save money for charging his PEV. Between the two, in most cases, the primary concern will be the travel time, especially in normal traffic conditions, when no discount is present. Nevertheless, the travel time aspect becomes less relevant if a heavy congestion arises; in this situation, the relative impact of the travel time spent at the CS decreases, and at the same time, the discounted energy price may steer the decision of the agent towards the choice of stopping to charge the PEV.

1) Number of vehicles: At each time interval $k \in \mathbb{N}$, the number of vehicles involved in the decision process is $n_{\mathrm{EV}}(k)$, which may vary due to the traffic conditions. From (2), we show that the total number of vehicles exiting cell 1 during $k$ is $\Phi_{1}^{-}(k) T$. Between those, the fraction of PEVs is denoted by $p_{\mathrm{EV}} \in[0,1]$. By relying on (2a) and (4), we attain

$$
\left\{\begin{array}{l}
n_{\mathrm{EV}}=\left\lfloor p_{\mathrm{EV}} D_{1} T\right\rfloor, \quad \text { if } D_{1}-\mathrm{r} 2 \mathrm{~s} \leq S_{2}-\mathrm{s} 2 \mathrm{r}  \tag{5}\\
n_{\mathrm{EV}}=\left\lfloor p_{\mathrm{EV}}\left(S_{2}-\mathrm{s} 2 \mathrm{r}+\mathrm{r} 2 \mathrm{~s}\right) T\right\rfloor, \quad \text { otherwise }
\end{array}\right.
$$

where the time dependency is omitted and $\lfloor x\rfloor \in \mathbb{N}$ denotes the floor of $x \in \mathbb{R}$.

To compute $n_{\mathrm{EV}}(k)$ via (5), the value of $\mathrm{r} 2 \mathrm{~s}(k)$ is necessary, even though it is the solution of the decision process that we are defining. Thus, it is not possible to exactly define $n_{\mathrm{EV}}(k)$. At the same time, $\operatorname{s} 2 \mathrm{r}(k)$ does not affect the computation of $n_{\mathrm{EV}}(k)$, since it is due to PEVs already at the CS, so not involved in the decision process arising at time $k$. To overcome this impasse, we compute the number of agents $n(k)$ involved in the game under the assumption of maximum congestion, namely if no agent stops at the charging station $(\mathrm{r} 2 \mathrm{~s}(k)=0)$. Then the number of agents taking part in the decision process is obtained as

$$
\left\{\begin{array}{l}
n(k)=\left\lfloor p_{\mathrm{EV}} D_{1}(k) T\right\rfloor, \quad \text { if } D_{1}(k)+\operatorname{s} 2 \mathrm{r}(k) \leq S_{2}(k) \\
n(k)=\left\lfloor p_{\mathrm{EV}}\left(S_{2}(k)-\mathrm{s} 2 \mathrm{r}(k)\right) T\right\rfloor, \quad \text { otherwise }
\end{array}\right.
$$

This assumption not only implies that $n(k)$ can be computed for every $k$, but also that all the vehicles involved in the game manage to exit cell 1 during the time interval $k$, and therefore being able to implement their strategies.
2) Decision variables and the SoC dynamics: The timevarying set indexing the $n(k)$ PEVs taking part in the game during the $k$-th time interval is denoted by $\mathcal{I}(k):=$ $\{1, \ldots, n(k)\}$. The decisions of the agents are performed over a prediction horizon $\mathcal{T}(k)$ of $T_{\mathrm{h}}$ time intervals. The length of the time intervals in the decision making process may be longer than the one used in the CTM. Specifically, we assume intervals of length $l T$, with $l \in \mathbb{N}$. Thus, the PEVs should plan their behavior over the set of intervals $\mathcal{T}(k):=$
$\left\{k, k+1, \ldots, k+T_{\mathrm{h}}\right\}$, where each index denotes an interval of length $l T$, e.g., $k \in \mathcal{T}(k)$ represents here $[k T, k T+l T)$ and similarly $k+1 \in \mathcal{T}(k)$ denotes $[k T+l T, k T+2 l T)$.

The State of Charge (SoC) of the battery of every PEV $i \in \mathcal{I}(k)$ at time $t \in \mathcal{T}(k)$ is denoted by $x_{i}(t) \in[0,1]$, where $x_{i}(t)=1$ represents a fully charged battery, while $x_{i}(t)=0$ a completely discharged one. The amount of energy purchased by agent $i \in \mathcal{I}(k)$ during the time period $t \in \mathcal{T}(k)$ is $u_{i}(t) \geq$ 0 . For every interval $t \in \mathcal{T}(k)$, the SoC reads as

$$
\begin{equation*}
x_{i}(t+1)=x_{i}(t)+b_{i} u_{i}(t) \tag{6}
\end{equation*}
$$

where $b_{i}=\frac{\eta_{i}}{C_{i}}>0$ is a coefficient associated to the battery efficiency $\eta_{i} \stackrel{i}{\in}(0,1]$ and capacity $C_{i}>0$. Let us introduce the variable $\delta_{i}(t) \in \mathbb{B}:=\{0,1\}$ as a binary decision variable, which takes value $\delta_{i}(t)=1$ if the vehicle is actively charging at the CS and $\delta_{i}(t)=0$ otherwise:

$$
\begin{equation*}
\left[\delta_{i}(t)=1\right] \Longleftrightarrow\left[u_{i}(t)>0\right], \quad \forall t \in \mathcal{T}(k), \forall i \in \mathcal{I}(k) \tag{7}
\end{equation*}
$$

The logical implication above entails that the energy purchased by PEV $i$ is positive if and only if $\delta_{i}(t)=1$, therefore we define $u_{i}(t) \in \mathcal{U}(t):=\left[\underline{u} \delta_{i}(t), \bar{u} \delta_{i}(t)\right]$, with $\bar{u}>0$ (respectively $\underline{u}>0$ ) being the maximum (minimum) energy that the vehicle can receive from the CS in a single time interval. We define a first collection of decision variables associated with each PEV $i \in \mathcal{I}(k)$, over the whole horizon $\mathcal{T}(k)$, as the collective vectors $u_{i}:=\operatorname{col}\left(\left(u_{i}(t)\right)_{t \in \mathcal{T}}\right), \delta_{i}:=\operatorname{col}\left(\left(\delta_{i}(t)\right)_{t \in \mathcal{T}}\right)$ and the evolution of the $\operatorname{SoC}, x_{i}:=\operatorname{col}\left(\left(x_{i}(t)\right)_{t \in \mathcal{T}}\right)$.
3) Travel time and congestion: An important quantity influencing the PEV decisions is the additional time $\xi(t) \geq 0$ that a PEV would experience due to the presence of congestion. Specifically, $\xi(t)$ denotes the difference between the travel time that a PEV experiences to actually travel throughout the cells $\{2, \ldots, N\}$ and the one it would spend in conditions of free flow. It provides insightful information on the traffic evolution, allowing the PEVs to discern whether or not they prefer to stop at the CS. If agent $i \in \mathcal{I}(k)$ decides to stop for charging, the congestion it will experience, when it merges back in the mainstream during the time interval $t_{i} \in \mathcal{T}(k)$, depends also on those that were behind it at the time of the decision $k<t_{i}$. Among all the vehicles exiting cell 1 in the time interval $\left(k, t_{i}\right]$, the PEVs have the possibility to stop for charging, deciding via a process akin to the one that agent $i$ is currently carrying out. For this reason, the exact value of $\xi(t)$ cannot be computed in advance by agent $i$ for the whole prediction horizon. We work around this difficulty by adopting a conservative approximation of $\xi(t)$, computed assuming that all the PEVs in $\mathcal{I}(k)$ and the ones following them do not stop at the CS. This leads to a value of $\xi(t)$ that over-estimates the actual experienced travel time. This approximation allows the agents to cope with the worst-case scenario, hence being able to meet possible time constraints. Moreover, it can be computed at every time interval and provides insightful information on the potential traffic evolution.

For a cell $\ell \in \mathcal{N}$, the vehicles' speed is attained as $v_{\ell}(k)=\Phi_{\ell}^{-}(k) / \rho_{\ell}(k)$. If an agent $i$ enters cell $\ell$ during the time interval $k$ and it takes $\bar{t} \in \mathbb{N}$ intervals to travel through it, then the velocity at which it will move when it enters the next
cell is $v_{\ell+1}(k+\bar{t})$. This observation motivates the following recursive, but implementable, definition

$$
\begin{equation*}
\xi(t):=\xi_{N}(t), \forall t \in \mathcal{T}(k) \tag{8}
\end{equation*}
$$

where

$$
\left\{\begin{array}{l}
\xi_{1}(t)=0 \\
\xi_{\ell}(t)=\xi_{\ell-1}(t)+\frac{L_{\ell}}{\hat{v}_{\ell}\left(t+\xi_{\ell-1}(t)\right)}-\frac{L_{\ell}}{\bar{v}_{\ell}}, \forall \ell\{2, \ldots, N\}
\end{array}\right.
$$

Here, $\hat{v}_{\ell}$ denotes the vehicles' speed computed in the worstcase scenario described above. The value $L_{\ell} / \bar{v}_{\ell}$ represents the travel time in the case of no congestion in the cell $\ell$. Under the assumption above, $\xi_{\ell}(t)$ can be always computed by letting the CTM evolve freely. It is worth noticing that, if the PEVs will experience no congestion along the whole freeway, i.e., $\hat{v}_{\ell}(t)=\bar{v}_{\ell}$ for all $\ell \in \mathcal{N}$, then $\xi(t)=0$.

Another important quantity related to the congestion is the number of vehicles that leave the CS at every time instant. In fact, if an agent leaves the CS when many others are also merging back into the mainstream, it may experience high levels of congestion. To model this phenomenon, we introduce a binary variable $\vartheta_{i} \in \mathbb{B}$ for every $i \in \mathcal{I}(k)$.
For every PEV $i \in \mathcal{I}(k)$ entering cell 2 at time $t_{i} \in \mathcal{T}(k)$, and for every $t \in \mathcal{T}(k)$, we define $\vartheta_{i}(t)=1$ if $t \in\left\{t_{i}-\right.$ $\left.W, \ldots, t_{i}+W\right\}$, and 0 otherwise. Thus, $\vartheta_{i}$ is a rectangular function of width at most $2 W+1$ intervals and at least $W+1$ (Figure 2). This variable is used to capture the influence of the PEVs entering cell 2 around the same time as agent $i$. The value of $W \in \mathbb{N}$ depends on $l T$. In fact, if $l T$ is large, then PEV $i$ will not experience the congestion due to the PEVs that precede or follow him, so $W=0$. On the other hand, if $l T$ is small, the value of $W$ has to be high to model correctly the possible congestion due to those agents that enter cell 2 around the same time as agent $i$. We elaborate further on this in the next section. Also in this case, we denote the collective vector over the whole $\mathcal{T}(k)$ as $\vartheta_{i}:=\operatorname{col}\left(\left(\vartheta_{i}(t)\right)_{t \in \mathcal{T}(k)}\right)$. Therefore, for every $t \in \mathcal{T}(k)$ and $i \in \mathcal{I}(k), \xi_{i}^{\mathrm{CS}}(t)$ approximates the extra time that agent $i$ would experience due to those PEVs entering cell 2 around time $t$ and it reads as

$$
\begin{equation*}
\xi_{i}^{\mathrm{CS}}(t):=\gamma\left(\sum_{\bar{k}<k} \sum_{j \in \mathcal{I}(\bar{k})} \vartheta_{j}(t)+\sum_{j \in \mathcal{I}(k) \backslash\{i\}} \vartheta_{j}(t)\right) . \tag{9}
\end{equation*}
$$

The first double summation, denoted by $\vartheta^{\text {old }}(t)$ for all $t \in$ $\mathcal{T}(k)$, represents the number of PEVs, that already completed the decision process, entering cell 2 during one of the intervals $\{\max (k, t-W), \ldots, t+W\}$. The coefficient $\gamma>0$ is proportional to the average amount of time agent $i$ spends for every PEV entering cell 2 during the intervals $\{\max (k, t-W), \ldots, t+W\}$. This coefficient may be estimated via historical data and engineering understanding or based on the worst-case scenario.
4) Energy price: In our model the dynamic energy price $p(k)$ is discounted by the HO in conjunction with a traffic congestion. On the other hand, it is also linked to the local energy demand required in the distribution network, i.e., $d(k)+u^{\mathrm{PEV}}(k)$, where $u^{\mathrm{PEV}}(k):=\sum_{\bar{k} \in \mathbb{N}} \sum_{i \in \mathcal{I}(\bar{k})} u_{i}(k)$ is the total energy purchased by the PEVs and $d(k)$ denotes the base


Fig. 2: Feasible choices of $\delta_{i}$ and $\vartheta_{i}$ for $i \in \mathcal{I}$ when $l=1$ and $W=1$. The two illustrations show when: (a) the PEV does not stop at the CS; (b) the PEV decides to stop.
energy demand of the local network. During the time interval $k$, we can study the congestion by looking at how much the travel time increases w.r.t. the free-flow case, for each cell $\ell \in \mathcal{N}$. This quantity is defined by $\Delta_{\ell}(k):=\frac{L_{\ell}}{v_{\ell}(k)}-\frac{L_{\ell}}{\bar{v}_{\ell}}$, and $\Delta_{\ell}(k) \geq 0$. Here we assume it is used by the HO to link the price to the congestion level, namely the higher $\Delta_{\ell}(k)$ the lower the price. Thus, the energy price that the HO imposes for every unit of energy purchased reads as

$$
\begin{equation*}
p(k):=c_{1} d(k)+c_{2} u^{\mathrm{PEV}}(k)-c_{3} \sum_{\ell=2}^{N} \Delta_{\ell}(k), \tag{10}
\end{equation*}
$$

where $c_{1}, c_{2}, c_{3}>0$ are scaling parameters tuned by the HO .
We note that the exact energy price applied in the future time intervals $t \in \mathcal{T}(k)$ cannot be computed in advance by the HO, since it depends on the traffic evolution, which is not completely known due to the arbitrary future choices of the drivers. Nevertheless, to allow the PEVs to perform an informed choice, we let the HO compute an estimation $\hat{p}(t)$ of the real $p(t)$ for the whole prediction horizon $\mathcal{T}(k)$. Then, this value is broadcast to the PEVs in $\mathcal{I}(k)$ and it is used by them to execute the decision process. If the congestion grows, then the price should drop, even though, intuitively, the discounted price leads to a larger number of PEVs stopping, and consequently an increment of $u^{\mathrm{PEV}}(k)$. We define $\hat{\Delta}_{\ell}(k)$ as an approximate value of $\Delta_{\ell}(k)$, which is computed by assuming that no agent exiting cell 1 during the prediction horizon $\mathcal{T}(k)$ stops at the CS. This assumption translates into $\mathrm{r} 2 \mathrm{~s}(t)=0$ for all $t \in \mathcal{T}(k)$. The density and the flow during the time interval $t \in \mathcal{T}(k)$ are attained by letting the CTM evolve freely. Therefore, the approximation of the additional time spent by the agent, due to the congestion in the cell $\ell$, reads as

$$
\begin{equation*}
\hat{\Delta}_{\ell}(t):=\frac{L_{\ell}}{\hat{v}_{\ell}(t)}-\frac{L_{\ell}}{\bar{v}_{\ell}} . \tag{11}
\end{equation*}
$$

The value of $\hat{\Delta}_{\ell}(t)$ overestimates the additional travel time spent due to the congestion level on the road during the
interval $t$. We introduce two time-varying vectors of offsets and coefficients, $\boldsymbol{\beta}_{0}(k):=\operatorname{col}\left(\left(\beta_{0}(t)\right)_{t \in \mathcal{T}(k)}\right)$ and $\boldsymbol{\beta}_{1}(k):=$ $\operatorname{col}\left(\left(\beta_{1}(t)\right)_{t \in \mathcal{T}(k)}\right)$ respectively, and define the estimated price, for every time interval $t \in \mathcal{T}(k)$, by

$$
\begin{equation*}
\hat{p}(t):=c_{1} d(t)-\left[\beta_{0}(t)+\beta_{1}(t) \sum_{\ell=2}^{N} \hat{\Delta}_{\ell}(t)\right] \tag{12}
\end{equation*}
$$

At every time instant $k \in \mathbb{N}$, the HO may use historical data on the traffic flow to compute the values of $\boldsymbol{\beta}_{0}(k)$ and $\boldsymbol{\beta}_{1}(k)$ that are supposed to minimize the error between the real and estimated price. This may be done with several techniques (e.g. linear regression or Bayesian estimation).

Remark 2: The definition in (12) implies that the estimated price is not affected by the strategies of the other agents in the game. Nevertheless, the strategies implemented by the PEVs involved during $k$, directly influence the estimated price used by the PEVs that will play the game during $k+1$. Therefore, the price dynamically changes over time and is assumed to be fixed only inside the single decision process.
5) Cost function formulation: The goal of each PEV is to find the best trade-off between saving money, and travel time. These two cost terms are described for every PEV $i \in \mathcal{I}(k)$ by the functions $J_{i}^{\text {price }}$ and $J_{i}^{\text {time }}$, respectively. The amount agent $i$ saves by charging at a discounted price depends on the total energy it purchases:

$$
J_{i}^{\mathrm{price}}\left(u_{i} \mid k\right):=\sum_{t \in \mathcal{T}(k)}\left(\hat{p}(t)-\bar{p}_{i}\right) u_{i}(t)
$$

where $\bar{p}_{i}>0$ represents the average cost agent $i$ would experience via standard fast charging, and it might vary between PEVs. Next, we define the cost associated to the total travel time experienced by vehicle $i$ by
$J_{i}^{\mathrm{time}}\left(\vartheta_{i}, \vartheta_{-i} \mid k\right):=\sum_{t \in \mathcal{T}(k)} \chi(t)\left[(t-k) v+\xi(t)+\xi_{i}^{\mathrm{CS}}(t)\right] \vartheta_{i}(t)$, where the notation $\boldsymbol{\vartheta}_{-i}$ is used to denote $\operatorname{col}\left(\left(\vartheta_{j}\right)_{j \in \mathcal{I}(k) \backslash\{i\}}\right)$. The quantity $(t-k)$ weights the time spent at the charging station, while the rest approximates the additional time spent if the agent enters cell 2 during the time interval $t$. The parameter $v>0$ weights the different perception that the agent has in spending time at the CS or in a congestion. The time-varying factor $\chi(t)$ normalizes the cost function with respect to the width of the rectangular function $\vartheta_{i}$. Note that the presence of $\xi_{i}^{\mathrm{CS}}(t)$ creates a coupling between all the decisions of the PEVs in the game. In $J_{i}^{\text {time }}$, the presence of $\vartheta_{i}$ entails that only some elements of the summation are not zero. Furthermore, its rectangular shape implies that the decision of agent $i$ depends also on those agents that enter cell 2 during an interval distant at most $2 W$ intervals from the one in which $i$ will enter cell 2 , see Figure 2 As anticipated, this feature models the different speeds of the vehicles in a cell.

Each agent may weight differently the two objectives thus we model the final cost as a convex combination of the two:

$$
\begin{equation*}
J_{i}\left(u_{i}, \vartheta_{i}, \boldsymbol{\vartheta}_{-i} \mid k\right):=\alpha_{i} J_{i}^{\text {price }}+\left(1-\alpha_{i}\right) J_{i}^{\text {time }} \tag{13}
\end{equation*}
$$

for some $\alpha_{i} \in(0,1)$. We study the effects of this parameter on the performance in [18](Part II: Case study). Finally, we
highlight that the nature of the approximation of $\xi_{i}^{\mathrm{CS}}$ and the estimation $\hat{p}$ are intrinsically different. In fact, despite both uncertainties are due to the presence of the human in the loop, the second is part of the policy designed by the HO to reduce the congestion, while the first is used to model the drivers' local decision. Consequently, the complete policy includes the actual price applied and its estimation over the prediction horizon, that are broadcast by the HO to the PEVs and used to influence their decision.

## B. Local and coupling constraints

We model the constraints on the drivers' possible choices as a collection of logical implications. First, we impose that the $2 W+1$ intervals in which $\vartheta_{i}=1$ are consecutive. To do so, we require that, for all $i \in \mathcal{I}(k), \vartheta_{i}$ changes its value from 0 to 1 and back to 0 only once:

$$
\begin{align*}
& \sum_{p \in \mathcal{T}(k)}\left(1-\vartheta_{i}(p-1)\right) \vartheta_{i}(p)=1  \tag{14a}\\
& \sum_{p \in \mathcal{T}(k)} \vartheta_{i}(p-1)\left(1-\vartheta_{i}(p)\right)=1 \tag{14b}
\end{align*}
$$

where $\vartheta_{i}(k-1)=0$, so the raising edge must precede the falling one. Then, we force the intervals in which $\vartheta_{i}(k)=1$ to be consecutive, and hence for all $i \in \mathcal{I}(k)$ and $t \in \mathcal{T}(k)$ it must hold that

$$
\begin{align*}
& \vartheta_{i}(t)\left(1-\vartheta_{i}(t+1)\right)\left(\sum_{p \in \mathcal{T}(k)} \vartheta_{i}(p)-\mathfrak{P}\right)=0  \tag{15}\\
& \mathfrak{P}:=\min \{t-k+W+1,2 W+1\} .
\end{align*}
$$

Clearly, if a PEV enters cell 2 at time $t_{i}$, it cannot charge in the remaining time intervals (Figure 24, and thus we obtain the following relation, for all $t \in \mathcal{T}(k)$ and $i \in \mathcal{I}(k)$ :

$$
\begin{array}{r}
{\left[\vartheta_{i}(t-1)\left(1-\vartheta_{i}(t)\right)=1\right] \Longrightarrow\left[\delta_{i}(r)=0\right]}  \tag{16}\\
\forall r \in\left\{t-W-1, \ldots, k+T_{\mathrm{h}}\right\}
\end{array}
$$

This condition models also the case in which a PEV is not stopping, so $\vartheta_{i}(k)=\cdots=\vartheta_{i}(k+W)=1$ and 16 implies that the PEV is never charging, i.e., $\delta_{i}(t)=0$ for all $t \in \mathcal{T}(k)$. Furthermore, (14) and (15) imply that the PEVs disconnect at least $W+1$ time intervals before the end of the prediction horizon. Next, we impose that, when an agent is done with charging, it exits the CS, and hence, for all $t \in \mathcal{T}(k)$, agent $i \in \mathcal{I}(k)$ has to satisfy

$$
\begin{align*}
{\left[\delta_{i}(t-1)=1\right] } & \wedge\left[\delta_{i}(t)=0\right] \Longrightarrow\left[\vartheta_{i}(r)=1\right] \\
\forall r & \in\{\max (k, t-W), \ldots, t+W\} \tag{17}
\end{align*}
$$

We impose that each PEV charges for at least $\bar{h} \in \mathbb{N}$ consecutive time intervals. In fact, the value $l T$ may be small and it is unreasonable to allow a PEV to stop for charging for only one time interval (e.g. 2 minutes). This translates into

$$
\begin{equation*}
\left[\delta_{i}(t-1)=0\right] \wedge\left[\delta_{i}(t)=1\right] \Rightarrow\left[\delta_{i}(t+h)=1, \forall h \leq \bar{h}\right] \tag{18}
\end{equation*}
$$

Similarly, if a PEV decides to stop, then we assume it remains at the CS for at least $2 W+1$ time intervals, and hence

$$
\begin{equation*}
\sum_{t \in \mathcal{T}(k)} \vartheta_{i}(t)>W+1 \Leftrightarrow \vartheta_{i}(k)=\cdots=\vartheta_{i}(k+W)=0 \tag{19}
\end{equation*}
$$

For each PEV, the minimum level of SoC necessary to reach the final destination from cell 2 is denoted by $x_{i}^{\text {ref }} \in(0,1]$. Thus, we assume that a PEV can enter cell 2 if $x_{i}(t)>x_{i}^{\text {ref }}$, otherwise it must stop (or remain) at the CS for charging, so for all $t \in \mathcal{T}(k)$,

$$
\begin{equation*}
\left[x_{i}(t)<x_{i}^{\mathrm{ref}}\right] \Longrightarrow\left[\vartheta_{i}(\max (k, t-W))=0\right] \tag{20}
\end{equation*}
$$

where $\vartheta_{i}(\max (k, t-W))=0$ implies that PEV $i$ cannot leave the charging station during the time interval $t$. The next constraint limits the maximum amount of energy that the CS can supply during each time interval by $u^{\max }>0$. Thus, for all $t \in \mathcal{T}(k)$, we have the following coupling constraint on the connected PEVs:

$$
\begin{equation*}
u^{\mathrm{old}}(t)+\sum_{i \in \mathcal{I}(k)} u_{i}(t) \leq u^{\max } \tag{21}
\end{equation*}
$$

where $u^{\text {old }}(t):=\sum_{\bar{k}<k} \sum_{j \in \mathcal{I}(\bar{k})} u_{j}(t)$ is the total energy that the agents, that already completed the decision process, planned to purchase during the time interval $t$.

Finally, we consider that if several PEVs stop at the CS simultaneously there can be a scarcity of charging plugs. Let $\bar{\delta}$ denote the total number of plugs at the CS. Then, we have

$$
\begin{equation*}
\delta^{\mathrm{old}}(t)-\sum_{i \in \mathcal{I}(k)} \delta_{i}(t)+\leq \bar{\delta}, \forall t \in \mathcal{T}(k) \tag{22}
\end{equation*}
$$

where $\delta^{\text {old }}(t)$ is defined analogously to $u^{\text {old }}(t)$.
The above constraints allow the PEVs to stop at the CS and do not start charging immediately (for example due to a lack of free plugs), and this may lead to the formation of a queue. We model the queue as a First-In-First-Out (FIFO), i.e., the vehicles already waiting have the priority over the PEVs entering it afterwards. This aspect is important to realistically model the PEV behaviors, which would be hard to formalize without the use of mixed-integer variables.

In Figure 2, we qualitatively represent a feasible choice of $\delta_{i}$ and $\vartheta_{i}$ for PEV $i$ and how it is reflected in the driver's behavior. In Figure 2a, agent $i$ does not stop at the CS. In comparison, in Figure 2p, the PEV enters the CS, but, since all the plugs are busy, it waits for the first two time intervals before connecting to the CS. Once it finishes the charging phase, i.e., $\delta_{i}(t)=0$, it merges back into the mainstream, according to 17).

We conclude this section by introducing a preliminary formulation of the set of inter-dependent mixed-integer optimization problems that model the decision process performed by the $n(k)$ PEVs during every time interval $k \in \mathbb{N}$ :

$$
\forall i \in \mathcal{I}(k):\left\{\begin{array}{cl}
\min _{u_{i}, x_{i}, \delta_{i}, \vartheta_{i}} & J_{i}\left(u_{i}, \vartheta_{i}, \boldsymbol{\vartheta}_{-i} \mid k\right)  \tag{P}\\
\text { s.t. } & \text { (6), } x_{i}(t) \in[0,1], \delta_{i}(t) \in \mathbb{B}, \\
& u_{i}(t) \in \mathcal{U}(t), \vartheta_{i}(t) \in \mathbb{B} \\
& (7), 14]-(22), \forall t \in \mathcal{T}(k)
\end{array}\right.
$$

Several constraints in $(\mathcal{P})$ are expressed via logical implications, thus this problem should be mathematically reformulated
to be solved. Specifically, in the Appendix we adopt a process akin to the one used in [12], [22] to transform the logical implications into mixed-integer affine coupling constraints by additional auxiliary variables.

## IV. Formulation of the mixed-integer game

As a result of translating the logical implications into affine constraints, we recast $(\overline{\mathcal{P}}$ as the following mixedinteger aggregative game, subject only to linear mixed-integer inequalities:

$$
\forall i \in \mathcal{I}(k):\left\{\begin{align*}
\min _{u_{i}, \ldots, \nu_{i}} & J_{i}\left(u_{i}, \vartheta_{i}, \vartheta_{i}, \boldsymbol{\vartheta}_{-i} \mid k\right)  \tag{k}\\
\text { s.t. } & x_{i}(t) \in[0,1], u_{i}(t) \in \mathcal{U}(t), \\
& \delta_{i}(t), \vartheta_{i}(t), \psi_{i}(t) \in \mathbb{B} \\
& \sigma_{i}(t), \omega_{i}(t) \in \mathbb{B} \\
& \varphi_{i}^{\mathrm{LH}}(t), \varphi_{i}^{\mathrm{HL}}(t), \nu_{i} \in \mathbb{B} \\
& \mu_{i}^{(h)}(t) \in \mathbb{B} \forall h \leq \bar{h} \\
& g_{i}(t), q_{i}(t) \in \mathbb{N} \\
& 21)-43), \forall t \in \mathcal{T}(k)
\end{align*}\right.
$$

The vector of all the decision variables in $\overline{\mathcal{G}(k)}$ is defined as

$$
\begin{aligned}
& z_{i}:=\operatorname{col}\left(u_{i}, x_{i}, \delta_{i}, \vartheta_{i}, \psi_{i}, \sigma_{i}, \varpi_{i}, \varphi_{i}^{\mathrm{LH}}\right. \\
& \left.\qquad \varphi_{i}^{\mathrm{HL}}, \mu_{i}^{(1)} \ldots, \mu_{i}^{(\bar{h})}, g_{i}, q_{i}, \nu_{i}\right) \in \mathbb{R}^{n_{i}},
\end{aligned}
$$

and $\boldsymbol{z}:=\operatorname{col}\left(\left(z_{i}\right)_{i \in \mathcal{I}(k)}\right)$, we obtain a compact form of $\mathcal{G}(k)$

$$
\begin{equation*}
\forall i \in \mathcal{I}(k): \quad \min _{z_{i} \in \mathcal{Z}_{i}(k)} J_{i}\left(z_{i}, \boldsymbol{z}_{-i} \mid k\right) \quad \text { s.t. } A \boldsymbol{z} \leq b \tag{23}
\end{equation*}
$$

where $\mathcal{Z}_{i}(k)$ is the set of strategy that satisfy the local constraints of $i$, while $A$ and $b$ are of suitable dimensions and are used to describe all the coupling constraints between the agents. We denote the set of all feasible strategies of player $i \in \mathcal{I}(k)$ as

$$
\begin{equation*}
\mathcal{Z}_{i}\left(\boldsymbol{z}_{-i} \mid k\right):=\left\{y \in \mathcal{Z}_{i}(k) \subset \mathbb{R}^{n_{i}} \mid A\left(y, \boldsymbol{z}_{-i}\right) \leq b\right\} \tag{24}
\end{equation*}
$$

where $\left(y, \boldsymbol{z}_{-i}\right)$ indicates the collective strategy vector, with $y$ being any feasible strategy of $i \in \mathcal{I}(k)$. Then, the set of all the feasible collective strategies is

$$
\mathcal{Z}(k):=\left\{\boldsymbol{z} \in \prod_{i \in \mathcal{I}(k)} \mathcal{Z}_{i}(k) \subset \mathbb{R}^{n} \mid A \boldsymbol{z} \leq b\right\}
$$

where $n:=\sum_{i \in \mathcal{I}(k)} n_{i}$.
Perhaps, the most popular notion of equilibrium for games like $\mathcal{G}(k)$ is the Generalized Nash Equilibrium (GNE), where no agent can reduce its cost by unilaterally changing its strategy to another feasible one [23], [24]. Here, we are interested in an approximate solution for mixed-integer games, i.e., $\varepsilon$-Mixed-Integer Nash Equilibrium ( $\varepsilon$-MINE).

Definition 1 ( $\varepsilon$-Mixed-Integer Nash equilibrium): A set of strategies $\boldsymbol{z}^{*} \in \mathcal{Z}$ is an $\varepsilon$-MINE, with $\varepsilon>0$, of the game $\mathcal{G}(k)$ if, for all $i \in \mathcal{I}$,

$$
J_{i}\left(z_{i}^{*}, \boldsymbol{z}_{-i}^{*} \mid k\right) \leq \inf _{z_{i} \in \mathcal{Z}_{i}\left(\boldsymbol{z}_{-i}^{*} \mid k\right)} J_{i}\left(z_{i}, \boldsymbol{z}_{-i}^{*} \mid k\right)+\varepsilon .
$$

with $\mathcal{Z}_{i}$ as in 24.

## A. Potential game structure

In this subsection, we prove that the game $\mathcal{G}(k)$ is an exact potential game [25]. Potential games are characterized by the existence of a potential function that describes the variation of the cost when an agent changes strategy.

Definition 2 (Exact potential function): A continuous function $P: \mathbb{R}^{n} \rightarrow \mathbb{R}$ is an exact potential function for the game $\mathcal{G}(k)$ if, for all $i \in \mathcal{I}(k)$, and $z_{i}, y_{i} \in \mathcal{Z}_{i}\left(\boldsymbol{z}_{-i} \mid k\right)$, it satisfies $J_{i}\left(z_{i}, \boldsymbol{z}_{-i} \mid k\right)-J_{i}\left(y_{i}, \boldsymbol{z}_{-i} \mid k\right)=P\left(z_{i}, \boldsymbol{z}_{-i}\right)-P\left(y_{i}, \boldsymbol{z}_{-i}\right)$.

To find the potential function $P$, we first reorganize the local cost function $J_{i}\left(z_{i}, \boldsymbol{z}_{-i} \mid k\right)$ as:

$$
\begin{equation*}
J_{i}\left(z_{i}, \boldsymbol{z}_{-i} \mid k\right)=\zeta_{i}\left(z_{i}\right)+\sum_{j \in \mathcal{I}(k) \backslash\{i\}} \lambda_{i, j}\left(z_{i}, z_{j}\right) \tag{25}
\end{equation*}
$$

where $\zeta_{i}$ depends on the local variables only, and $\lambda_{i, j}$ incorporates the cross terms depending on the other players' strategy $z_{j}$. From (9), we derive that

$$
\begin{equation*}
\lambda_{i, j}\left(z_{i}, z_{j}\right):=\sum_{t \in \mathcal{T}(k)} \chi(t) \gamma \vartheta_{j}(t) \vartheta_{i}(t) \tag{26}
\end{equation*}
$$

Thus, $\lambda_{i, j}\left(z_{i}, z_{j}\right)=\lambda_{j, i}\left(z_{j}, z_{i}\right)$ meaning that the agents influence each other in a symmetric way. In the next statement, we introduce the exact potential function for the game in $\mathcal{G}(k)$

Theorem 1: For each $k \in \mathbb{N}$, the game $\mathcal{G}(k)$ is an exact potential game with

$$
P(\boldsymbol{z} \mid k):=\sum_{i \in \mathcal{I}(k)}\left(\zeta_{i}\left(z_{i}\right)+\sum_{j \in \mathcal{I}(k), j<i} \lambda_{i, j}\left(z_{i}, z_{j}\right)\right)
$$

as an exact potential function, where $\lambda_{i, j}$ is as in 26.
Proof: The proof is akin to the one in [26].
The pivotal result that highlights the importance of the above theorem is that an $\varepsilon$-approximated minimum of the potential function is also an $\varepsilon$-MINE of the game $\mathcal{G}(k)$, see [27, Th. 2]. Thus, it is sufficient to show that the proposed algorithm converges to a minimum of the potential function in order to achieve the sought convergence result.

## V. CTM TRAFFIC CONTROL SCHEME

We can now focus on the connection between the traffic dynamics and the decision process of the PEVs. Then, we describe in details our proposed algorithm that the agents can use to seek an equilibrium of the game.

## A. Iterative semi-decentralized algorithm

We propose here a semi-decentralized iterative algorithm (Algorithm 1) that the agents in $\mathcal{I}(k)$ can adopt to solve the Mixed-Integer Generalized Potential Game (MI-GPG) $\mathcal{G}(k)$ The notation $z_{i}(\tau)$ denotes the strategy of agent $i$ at the $\tau$-th iteration of the algorithm.

After the initialization step, where the players receive the information broadcast by the HO, each PEV decides to update its strategy independently from the others. If an agent wants to update, it sends a request to the HO. If no other player is currently updating, then agent $i$ starts its local update given the aggregate quantities $\sum_{j \in \mathcal{I}(k) \backslash\{i\}} \vartheta_{j}(t), \sum_{j \in \mathcal{I}(k) \backslash\{i\}} u_{i}(t)$
and $\sum_{j \in \mathcal{I}(k) \backslash\{i\}} \delta_{i}(t)$, used to compute the cost $J_{i}^{\text {time }}$ and the coupling constraints 21, 22. On the other hand, if another agent is performing the update, agent $i$ enters a FIFO queue from which the HO extracts sequentially the future agents that are allowed to update. At the moment of the update, agent $i$ computes a best-response strategy $z_{i}^{*}$ w.r.t. the strategies of the others. We define the mixed-integer best-response mapping for agent $i \in \mathcal{I}(k)$, as

$$
\mathcal{B R}_{i}\left(\boldsymbol{z}_{-i}\right):=\left\{\begin{array}{l}
\operatorname{argmin}_{z_{i}} J_{i}\left(z_{i}, \boldsymbol{z}_{-i} \mid k\right)  \tag{27}\\
\quad \text { s.t. }\left(z_{i}, \boldsymbol{z}_{-i} \mid k\right) \in \mathcal{Z}(k)
\end{array}\right.
$$

where $\mathcal{B} \mathcal{R}_{i}$ may be a set, thus $z_{i}^{*} \in \mathcal{B R}_{i}\left(\boldsymbol{z}_{-i}\right)$.
Agent $i$ updates its current strategy only if $z_{i}^{*}$ leads to an (at least) $\varepsilon$-improvement in terms of minimization of its cost.

The iteration is completed after the PEV communicates to the HO its (possibly) new strategy and the HO uses it to revise all the quantities in the game that depend on $z_{i}$.

In the following result, we show that Algorithm 1 converges to an $\varepsilon$-MINE of the game $\mathcal{G}(k)$, under the assumption that all the players manage to update their strategies over a sufficiently large number of iterations.

Proposition 1: Let $\varepsilon>0$ and $k \in \mathbb{N}$, and assume that for every $j \in \mathcal{I}(k)$ and $\tau \in \mathbb{N}$ there exists a $\bar{\tau}>\tau$ such that $j \in\{i(\tau), \ldots, i(\bar{\tau})\}$. Then, Algorithm 1 computes an $\varepsilon$-MINE of the game $\mathcal{G}(k)$ in (23).

Proof: From Theorem 1, $\mathcal{G}(k)$ is an MI-GPG with an exact potential function for all $k \in \mathbb{N}$. Therefore, the result in [27. Th. 4] applies to show that the sequential best-response based algorithm proposed in Algorithm 1 converges to an $\varepsilon$ MINE of the game.

Remark 3 (Privacy and scalability): In Algorithm 1, the HO shares with each PEV only aggregate information on the choices of the others. This feature allows to preserve the privacy of the agents in the game, since an agent cannot retrieve the local decision strategy of another PEV based on the data received from the HO. Moreover, using aggregate information is also important to preserve the scalability of Algorithm 11 In fact, the amount of data shared between each PEV and the HO does not grow with $n(k)$. This is crucial to obtain an implementable solution, due to the (possibly) large number of vehicles involved.

## B. Complete CTM control loop

The HO, introduced in Section III plays a crucial role in collecting and broadcasting information from and to the vehicles on the highway stretch. We propose the following decision process which takes place at the beginning of every time interval $k \in \mathbb{N}$ via the following four steps.
S.1) HO collects information: The HO collects information, from the sensors on the highway (placed at the interfaces between cells), on the cells' density, i.e., $\rho_{\ell}(k)$ for all $\ell \in \mathcal{N}$. The HO computes the following set of variables: $\xi(t)$ via (8), $\vartheta^{\text {old }}(t), \delta^{\text {old }}(t), u^{\text {old }}(t), \Delta_{\ell}(k), \hat{\Delta}_{\ell}(t)$ via 11, $p(k)$ via 10) and $\hat{p}(t)$ via (12), by exploiting the CTM and the strategies of the PEVs that performed the process during the previous time intervals.

```
Algorithm 1: Sequential best-response
Initialization: For \(k \in \mathbb{N}\), HO sends to every \(i \in \mathcal{I}(k)\)
    the coefficients \(\bar{h}, u^{\max }, \bar{\delta}, \gamma \in \mathbb{R}\) and \(\xi(t)\),
    \(\vartheta^{\text {old }}(t), u^{\text {old }}(t), \delta^{\text {old }}(t), \hat{p}(t), \forall t \in \mathcal{T}(k)\).
    Update: Choose \(\boldsymbol{z}(0) \in \mathcal{Z}(k)\), set \(\tau:=0\)
    while \(\boldsymbol{z}(\tau)\) is not an \(\varepsilon\)-MINE do
        HO do
            Extracts from the waiting queue
                \(i:=i(\tau) \in \mathcal{I}(k)\).
            Sends \(\sum_{j \in \mathcal{I}(k) \backslash\{i\}} \vartheta_{j}(t), \sum_{j \in \mathcal{I}(k) \backslash\{i\}} u_{i}(t)\) and
                \(\sum_{j \in \mathcal{I}(k) \backslash\{i\}} \delta_{i}(t)\) to \(i\).
    end
        Player \(i\) do
            Computes \(z_{i}^{*}(\tau) \in \mathcal{B} \mathcal{R}_{i}\left(\boldsymbol{z}_{-i}(\tau)\right)\) as in (27)
            if \(\left.\left.J_{i}\left(z_{i}(\tau), \boldsymbol{z}_{-i}(\tau)\right)\right)-J_{i}\left(z_{i}^{*}(\tau), \boldsymbol{z}_{-i}(\tau)\right)\right) \geq \varepsilon\)
                \(z_{i}(\tau+1):=z_{i}^{*}(\tau)\)
            else
                \(z_{i}(\tau+1):=z_{i}(\tau)\)
            end
            Sends \(z_{i}(\tau+1)\) to HO
        end
        Set \(z_{j}(\tau+1):=z_{j}(\tau) \forall j \neq i, \tau:=\tau+1\)
    end
```

S.2) HO broadcasts information: Those PEVs that have the possibility to stop during the time interval $k$, i.e., the ones leaving cell 1 , connect with the HO , forming the set $\mathcal{I}(k)$ of players involved in the game. The HO broadcasts to all of them the quantities they need to initialize the game $\mathcal{G}(k)$, i.e., the initialization phase in Algorithm 1 Moreover, the HO applies the price $p(k)$ in (10) to the energy purchased by the PEVs currently charging at the CS.
S.3) Iterative solution of the decision process: After the initialization, the agents update their strategy as shown in Algorithm 1, and described in Section V-A The PEVs keep updating until they converge to an $\varepsilon$-MINE of the game $\mathcal{G}(k)$, hence a feasible set of strategies $\boldsymbol{z} \in \mathcal{Z}(k)$, which is convenient to each of the PEVs. We stress that the iterations $\tau \in \mathbb{N}$ to solve the game are unrelated to the intervals of the CTM or the intervals in $\mathcal{T}(k)$, and in fact they are all completed within the interval $k$.
S.4) Strategy implementation: The agents in $\mathcal{I}(k)$ implement their final strategies (i.e., stop at the CS or continue driving) and the process will start again from (S.1) at the beginning of the interval $k+1$.

The presence of the human in the loop imposes a bi-level implementation of step (S.4). We envision that every PEV performs the computations in (S.3) via a dedicated software, then the final strategy is translated into a simple message that is prompted to the human user advising whether it is convenient or not to stop at the CS. In the end, the driver implements the suggested behavior.

Finally, we want to elaborate on how to compute, starting from $\boldsymbol{z}$, the in and out flow of the CS, i.e., r2s and s 2 r respectively. From the constrains in Sec. III-B, agent $i \in \mathcal{I}(k)$
does not stop at the CS if and only if $\vartheta_{i}(k)=1$, thus the flow entering the CS is defined by

$$
\begin{equation*}
\mathrm{r} 2 \mathrm{~s}(k):=\frac{1}{T}\left(n(k)-\sum_{i \in \mathcal{I}(k)} \vartheta_{i}(k)\right) \tag{28}
\end{equation*}
$$

The flow exiting the CS is

$$
\begin{equation*}
\operatorname{s2r}(k):=\frac{1}{l T} \sum_{\bar{k}<k} \sum_{j \in \mathcal{I}(\bar{k})} \vartheta_{j}(k-W) \vartheta_{j}(k+W) \tag{29}
\end{equation*}
$$

where the double summation selects only those agents exiting the CS during the time interval $k$. In fact, for those PEVs that do not stop, we have $\sum_{t \in \mathbb{N}} \vartheta_{j}(t)=W+1$, thus they never contribute to (29). Conversely, if PEV $i$ stops, $\vartheta_{j}(k-W) \vartheta_{j}(k+W)=1$ if $i$ exists the CS during $k$. The PEVs that contribute to $\mathrm{s} 2 \mathrm{r}(k)$ decide to exit during an interval $m$ long $l T$ time instants that encloses $k$, so the contribution to the flow is $1 / l$. The definitions above represent the actual connections between the CTM and the decision process, thus we have arrived at the goal stipulated at the beginning of the paper.

## VI. Conclusion

On a highway stretch with one charging station, the adoption of a dynamic energy price, discounted proportionally to the traffic level, can contribute to alleviating the traffic congestion. It incentivizes the owners of plug-in electric vehicles to stop for charging during, or close to, rush hours. Under the assumption of a rational self-interested behavior, a multiagent game arises between the plug-in electric vehicles that have to choose whether or not it is convenient to stop at the charging station. The decision process can be formalized as a mixed-integer generalized potential game, and solved via a semi-decentralized iterative scheme, where the highway operator acts as an aggregator. This control scheme converges to a mixed-integer Nash equilibrium of the game, i.e., an approximated optimal charging strategy for the electric vehicles that alleviates the traffic congestion. The effectiveness of our methodology is shown in [18 via numerical simulations with real-world data.

This work is the first that proposes a tight integration of traffic dynamics and charging incentives. For this reason, it may be the cornerstone of several further developments. Some assumptions may be relaxed, and the same idea can be applied to other traffic models, considering also agents that are not perfectly rational drivers, e.g. using relative best response dynamics [28]. Finally, the case of several charging stations and ramps is left as future work.

## REFERENCES

[1] A. Schroten, H. van Essen, L. van Wijngaarden, D. Sutter, and E. Andrew, "Sustainable transport infrastructure charging and internalisation of transport externalities: Executive summary," European Commission, Tech. Rep., May 2019.
[2] M. Barth and K. Boriboonsomsin, "Traffic congestion and greenhouse gases," Access Magazine, vol. 1, no. 35, pp. 2-9, 2009.
[3] A. A. Ganin, M. Kitsak, D. Marchese, J. M. Keisler, T. Seager, and I. Linkov, "Resilience and efficiency in transportation networks," Science Advances, vol. 3, no. 12, 2017.
[4] S. Cairns, L. Sloman, C. Newson, J. Anable, A. Kirkbride, and P. Goodwin, "Smarter choices: Assessing the potential to achieve traffic reduction using 'soft measures'," Transport Reviews, vol. 28, no. 5, pp. 593-618, 2008.
[5] G. Nilsson, P. Hosseini, G. Como, and K. Savla, "Entropy-like lyapunov functions for the stability analysis of adaptive traffic signal controls," in 54th IEEE Conference on Decision and Control, 2015, pp. 2193-2198.
[6] Q. Guo, L. Li, and X. J. Ban, "Urban traffic signal control with connected and automated vehicles: A survey," Transportation Research Part C: Emerging Technologies, vol. 101, pp. 313 - 334, 2019.
[7] G. Piacentini, P. Goatin, and A. Ferrara, "Traffic control via moving bottleneck of coordinated vehicles," IFAC-PapersOnLine, vol. 51, no. 9, pp. 13-18, 2018.
[8] A. Downs, Still stuck in traffic: coping with peak-hour traffic congestion. Brookings Institution Press, 2005.
[9] W. Riggs, "Painting the fence: Social norms as economic incentives to non-automotive travel behavior," Travel Behaviour and Society, vol. 7, pp. $26-33,2017$.
[10] X. Hu, Y.-C. Chiu, and L. Zhu, "Behavior insights for an incentivebased active demand management platform," International Journal of Transportation Science and Technology, vol. 4, no. 2, pp. 119 - 133, 2015.
[11] E. Ben-Elia and D. Ettema, "Changing commuters' behavior using rewards: A study of rush-hour avoidance," Transportation Research Part F: Traffic Psychology and Behaviour, vol. 14, no. 5, pp. 354-368, 2011.
[12] C. Cenedese, F. Fabiani, M. Cucuzzella, J. M. A. Scherpen, M. Cao, and S. Grammatico, "Charging plug-in electric vehicles as a mixed-integer aggregative game," in 58th IEEE Conference on Decision and Control, 2019, pp. 4904-4909.
[13] Z. Ma, D. S. Callaway, and I. A. Hiskens, "Decentralized charging control of large populations of plug-in electric vehicles," IEEE Transactions on Control Systems Technology, vol. 21, no. 1, pp. 67-78, 2013.
[14] F. Parise, M. Colombino, S. Grammatico, and J. Lygeros, "Mean field constrained charging policy for large populations of plug-in electric vehicles," in 53rd IEEE Conference on Decision and Control, 2014, pp. 5101-5106.
[15] Y. Xu, S. Çolak, E. C. Kara, S. J. Moura, and M. C. González, "Planning for electric vehicle needs by coupling charging profiles with urban mobility," Nature Energy, vol. 3, pp. 484-493, 2018.
[16] W. Jing, M. Ramezani, K. An, and I. Kim, "Congestion patterns of electric vehicles with limited battery capacity," PLOS ONE, vol. 13, no. 3, pp. 1-18, 032018.
[17] V. del Razo and H. Jacobsen, "Smart charging schedules for highway travel with electric vehicles," IEEE Transactions on Transportation Electrification, vol. 2, no. 2, pp. 160-173, 2016.
[18] C. Cenedese, M. Cucuzzella, J. Scherpen, S. Grammatico, and M. Cao, "Highway Traffic Control via Smart e-Mobility - Part II: Dutch A13 Case Study," IEEE-Transaction on intelligent transportation systems (submitted), 2021.
[19] C. F. Daganzo, "The cell transmission model, part ii: Network traffic," Transportation Research Part B: Methodological, vol. 29, no. 2, pp. 79 - 93, 1995.
[20] A. Ferrara, S. Sacone, and S. Siri, Freeway traffic modelling and control. Springer, 2018.
[21] J. Neubauer and E. Wood, "The impact of range anxiety and home, workplace, and public charging infrastructure on simulated battery electric vehicle lifetime utility," Journal of Power Sources, vol. 257, pp. 12 - 20, 2014.
[22] F. Fabiani and S. Grammatico, "Multi-vehicle automated driving as a generalized mixed-integer potential game," IEEE Transactions on Intelligent Transportation Systems, 2019, (In press).
[23] C. Cenedese, G. Belgioioso, Y. Kawano, S. Grammatico, and M. Cao, "Asynchronous and time-varying proximal type dynamics in multi-agent network games," IEEE Transactions on Automatic Control, pp. 1-1, 2020.
[24] C. Cenedese, G. Belgioioso, S. Grammatico, and M. Cao, "An asynchronous, forward-backward, distributed generalized nash equilibrium seeking algorithm," in 18th European Control Conference, 2019, pp. 3508-3513.
[25] D. Monderer and L. S. Shapley, "Potential games," Games and economic behavior, vol. 14, no. 1, pp. 124-143, 1996.
[26] F. Fabiani and A. Caiti, "Nash equilibrium seeking in potential games with double-integrator agents," in 2019 18th European Control Conference (ECC). IEEE, 2019, pp. 548-553.
[27] S. Sagratella, "Algorithms for generalized potential games with mixedinteger variables," Computational Optimization and Applications, pp. 1-29, 2017.
[28] A. Govaert, C. Cenedese, S. Grammatico, and M. Cao, "Relative best response dynamics in finite and convex network games," in 58th IEEE Conference on Decision and Control, 2019, pp. 3134-3139.

## Appendix

## A. Translate logical constraints into mixed-integer affine constraints

First, we show how to translate the basic types of logical implications in sets of inequalities by means of auxiliary variables. Given a linear function $f: \mathbb{R} \rightarrow \mathbb{R}$, let us define $M:=\max _{x \in \mathcal{X}} f(x), m:=\min _{x \in \mathcal{X}} f(x)$ where $\mathcal{X}$ is a compact set. Then, for $c \in \mathbb{R}$ and $\varphi \in \mathbb{B}$, the triplet $f, \varphi, c$ satisfy the implication $[\varphi=1] \Longleftrightarrow[f(x) \geq c]$ if and only if it is a solution of the set of mixed-integer inequalities $\mathcal{S}_{\geq}$, defined as

$$
\mathcal{S}_{\geq}(\varphi, f(x), c):=\left\{\begin{array}{l}
(c-m) \varphi \leq f(x)-m \\
(M-c+\epsilon) \varphi \geq f(x)-c+\epsilon
\end{array}\right.
$$

The value of $\epsilon>0$ represents a small tolerance on the constraint violation. Similarly, the condition $[\varphi=1] \Longleftrightarrow$ $[f(x) \leq c]$ is translated into:

$$
\mathcal{S}_{\leq}(\varphi, f(x), c):=\left\{\begin{array}{l}
(M-c) \varphi \leq M-f(x) \\
(c+\epsilon-m) \varphi \geq \epsilon+c-f(x)
\end{array}\right.
$$

The logical AND between two binary variables $\sigma, \tau \in \mathbb{B}$, i.e, $[\varphi=1] \Longleftrightarrow[\sigma=1] \wedge[\tau=1]$ is equivalent to:

$$
\mathcal{S}_{\wedge}(\varphi, \sigma, \tau):=\left\{\begin{aligned}
&-\sigma+\varphi \leq 0 \\
&-\tau+\varphi \leq 0 \\
& \sigma+\tau-\varphi \leq 1
\end{aligned}\right.
$$

Finally, the product of binary and continuous variables can be transformed into mixed-integer linear inequalities as follows:
$\mathcal{S}_{\Rightarrow}(g, f(x), \varphi):=\left\{\begin{array}{l}m \varphi \leq g \leq M \varphi \\ -M(1-\varphi) \leq g-f(x) \leq-m(1-\varphi)\end{array}\right.$
The latter is equivalent to: $[\varphi=0] \quad \Longrightarrow \quad[g=0]$, while $[\varphi=1] \Longrightarrow[g=f(x)]$.

1) Translate the logical implications in $\overline{\mathcal{P}}$ : The expression in (7) can be directly transformed into the set of linear inequalities

$$
\begin{equation*}
\mathcal{S}_{\geq}\left(\delta_{i}(t), u_{i}(t), \underline{u}\right) \tag{30}
\end{equation*}
$$

The two conditions in can be recast using the two auxiliary variables $\varphi_{i}^{\mathrm{LH}}, \varphi_{i}^{\mathrm{HL}} \in \mathbb{B}$ denoting the rising and falling edge of $\vartheta_{i}$ respectively, so, for every $t \in \mathcal{T}(k)$, they have to satisfy

$$
\begin{align*}
& \mathcal{S}_{\wedge}\left(\varphi_{i}^{\mathrm{LH}}(t),\left(1-\vartheta_{i}(t-1)\right), \vartheta_{i}(t)\right),  \tag{31a}\\
& \mathcal{S}_{\wedge}\left(\varphi_{i}^{\mathrm{HL}}(t), \vartheta_{i}(t-1),\left(1-\vartheta_{i}(t)\right)\right) . \tag{31b}
\end{align*}
$$

Using the variable introduced above for all $t \in \mathcal{T}(k)$, the condition in (15) becomes

$$
\begin{aligned}
& \varphi_{i}^{\mathrm{HL}}(t+1) g_{i}(t)=0 \\
& g_{i}(t):=\sum_{p \in \mathcal{T}(k)} \vartheta_{i}(p)-\min \{t-k+W+1,2 W+1\}
\end{aligned}
$$

The nonlinearity in (32) can be converted using the auxiliary variable $q_{i}(t)$ for every $t \in \mathcal{T}(k)$ as

$$
\begin{equation*}
\mathcal{S}_{\Rightarrow}\left(q_{i}(t), g_{i}(t), \varphi_{i}^{\mathrm{HL}}(t+1)\right) \tag{33}
\end{equation*}
$$

Finally, (32) becomes

$$
\begin{equation*}
q_{i}(t)=0 \tag{34}
\end{equation*}
$$

The constraint in 16 is equivalent to the linear inequality

$$
\begin{equation*}
\sum_{p=k}^{t+W+1} \varphi_{i}^{\mathrm{HL}}(p)+\delta_{i}(t) \leq 1, \forall t \in \mathcal{T}(k) . \tag{35}
\end{equation*}
$$

The logical implication (17) requires the introduction of an auxiliary binary variable $\psi_{i}(t) \in \mathbb{B}$ defined for every $t \in$ $\mathcal{T}(k)$ as $\left[\psi_{i}(t)=1\right] \Longleftrightarrow\left[\delta_{i}(t-1)=1\right] \wedge\left[\delta_{i}(t)=0\right]$. This implication can be transformed to the set of inequalities

$$
\begin{equation*}
\mathcal{S}_{\wedge}\left(\psi_{i}(t), \delta_{i}(t-1),\left(1-\delta_{i}(t)\right)\right) . \tag{36}
\end{equation*}
$$

Then, (17) translates into $\left[\psi_{i}(t)=1\right] \Longrightarrow\left[\vartheta_{i}(r)=1\right]$ that can be rephrased, for all $t \in \mathcal{T}(k)$ and $r \in\{\max (k, t-$ $W), \ldots, t+W\}$, as

$$
\begin{equation*}
\vartheta_{i}(r)-\psi_{i}(t) \geq 0 \tag{37}
\end{equation*}
$$

The constraint in 18 requires an additional step. First, we introduce the binary auxiliary variable $\left[\sigma_{i}(t)=1\right] \Longleftrightarrow$ $\left[\delta_{i}(t-1)=0\right] \wedge\left[\delta_{i}(t)=1\right]$, for every $t \in \mathcal{T}(k)$. By exploiting $\sigma_{i}$, 18) can be equivalently written as

$$
\begin{equation*}
\sigma_{i}(t)\left(\sum_{h=1}^{\bar{h}} \delta_{i}(t+h)-\bar{h}\right)=0 \tag{38}
\end{equation*}
$$

Next, to eliminate the nonlinearity in 38, we define $\bar{h}$ auxiliary binary variables $\mu_{i}^{(1)}(t), \ldots, \mu_{i}^{(\bar{h})}(t) \in \mathbb{B}$ for every $t$ as

$$
\begin{equation*}
\mathcal{S}_{\wedge}\left(\mu_{i}^{(h)}(t), \sigma_{i}(t), \delta_{i}(t+h)\right), \forall h \in\{1, \ldots, \bar{h}\} \tag{39}
\end{equation*}
$$

Thus, 38 reduces to the linear equation

$$
\begin{equation*}
\sum_{h=1}^{\bar{h}} \mu_{i}^{(h)}(t)-\sigma_{i}(t) \bar{h}=0 \tag{40}
\end{equation*}
$$

We transform (19) into the following set of linear constraints by introducing the auxiliary variable $\nu_{i} \in \mathbb{B}$, and hence for every $i \in \mathcal{I}(k)$ it must be true that

$$
\begin{array}{r}
0<\nu_{i}+\frac{1}{W+1} \sum_{p=0}^{W} \vartheta_{i}(k+p) \leq 1  \tag{41}\\
\mathcal{S}_{\geq}\left(\nu_{i}, \sum_{t \in \mathcal{T}(k)} \vartheta_{i}(t), W+2\right)
\end{array}
$$

The last logical implication to be transformed into a linear inequality is 20. The variable $\omega_{i}(t) \in \mathbb{B}$ is defined as $\left[x_{i}(t)<x_{i}^{\text {ref }}\right] \Longleftrightarrow[\omega(t)=1]$, so it satisfies the pattern of inequalities

$$
\begin{equation*}
\mathcal{S}_{\leq}\left(\omega_{i}(t), x_{i}(t), x_{i}^{\mathrm{ref}}\right) \tag{42}
\end{equation*}
$$

Finally, 20) is equivalent to the linear inequality

$$
\begin{equation*}
\omega_{i}(t)+\vartheta_{i}((\max (k, t-W))) \leq 1 \tag{43}
\end{equation*}
$$



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