Event-Triggered Second Order Sliding Mode Control of Nonlinear Uncertain Systems

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Abstract— **This paper presents a novel Second Order Sliding Mode (SOSM) control algorithm for a class of nonlinear systems subject to matched uncertainties. By virtue of its Event-Triggered nature, it can be used as a basis to construct robust networked control schemes. The algorithm objective is indeed to reduce the number of state transmissions over the network, in order to alleviate the network congestion and reduce possible packet loss, jitter and delays, while guaranteeing satisfactory performance in terms of stability and robustness. The proposed Event-Triggered Second Order Sliding Mode control strategy is theoretically analyzed in the paper, showing its capability of enforcing the robust ultimately boundedness of the sliding variable and its first time derivative, and consequently the practical stability of the uncertain nonlinear system, in spite of the significant reduction of the number of state transmissions with respect to a conventional SOSM control approach. The satisfactory performance of the proposed scheme are also assessed in simulation.**

I. Introduction

In Networked Control Systems (NCSs), the presence of the network in the control loop can determine a decrement of the performance because of packet loss, jitter, and transmission delays, which can affect the overall system [1]. Moreover, the network malfunctions tend to increase with the network congestion, so that the design of robust control schemes, able to reduce the measurements transmissions over the network, can be beneficial.

In the literature, the so-called Event-Triggered (ET) control approach [2] has been proposed as an effective solution for networked control schemes. In contrast to conventional time-triggered control, which features periodic transmissions of the state measurements, ET control enables the state measurements transmissions only when a pre-specified triggering condition is satisfied (or violated, depending on the adopted logic). This significantly reduces the transmissions over the network and the consequent possible network congestion caused by the control activity. Moreover, in the literature, it has been proved that, in spite of the aperiodic transmission of the state of the controlled system, satisfactory stability properties can be guaranteed by ET control schemes [2], [3].

On the other hand, Sliding Mode (SM) control is a well-known robust control approach, especially useful to control systems subject to modelling uncertainties and external disturbances [4], [5]. Because of its robustness, SM control is an effective control strategy also in case of NCSs. The same holds for higher order and, in particular, Second Order Sliding Mode (SOSM) control [6]–[8], in which not only the sliding variable but also some of its time derivatives are steered to zero in a finite time. This is confirmed by the numerous applications described in the literature (see, for instance, [9]–[16]).

In this paper, SOSM control and ET control are coupled to design a control scheme, which can be useful in networked implementations. More precisely, we do not adopt a mathematical model of the network, but we design the control strategy in order to reduce data transmission as much as possible. The proposed ET-SOSM control algorithm, designed for a rather general class of uncertain nonlinear systems, is based on a triggering condition which depends on the sliding variable and its first time derivative. The considered system subject to the application of the proposed algorithm is theoretically analyzed in the paper, proving the ultimately boundedness, in a suitable convergence set, of the sliding variable and its first time derivative, and consequently the practical stability of the uncertain nonlinear system. This practical stability result is obtained in spite of the significant reduction of the number of state transmissions with respect to a conventional SOSM control approach.

Note that in previous papers [17], [18], the combined use of SM control and Model-Based ET or genuine ET control was already discussed. The differences between the present paper and [17] lie in the possibility of avoiding the use of a nominal model, of solving a sliding mode control problem for systems with relative degree 2, and of providing an intrinsic chattering alleviation capability in case of systems with relative degree 1.

The present paper is organized as follows. In Section II, the considered control problem is suitably formulated. The proposed control strategy is described in details in Section III. In Section IV, the stability properties of the overall ET-SOSM control scheme are formally analyzed, while some simulation results are reported and discussed in Section V. Some conclusions are finally gathered in Section VI.

II. Problem Formulation

Consider a plant (process and actuator) which can be described by the single-input system affine in the control variable

$$\dot{x}(t) = a(x(t)) + b(x(t))u(t) + d_{\mathrm{m}}(x(t),t) \tag{1}$$

where $x \in \Omega$ ($\Omega \subset \mathbb{R}^n$ bounded) is the state vector, the value of which at the initial time instant t_0 is $x(t_0) = x_0$, and $u \in \mathbb{R}$ is a scalar input subject to the saturation $[-U_{\max}, U_{\max}]$,

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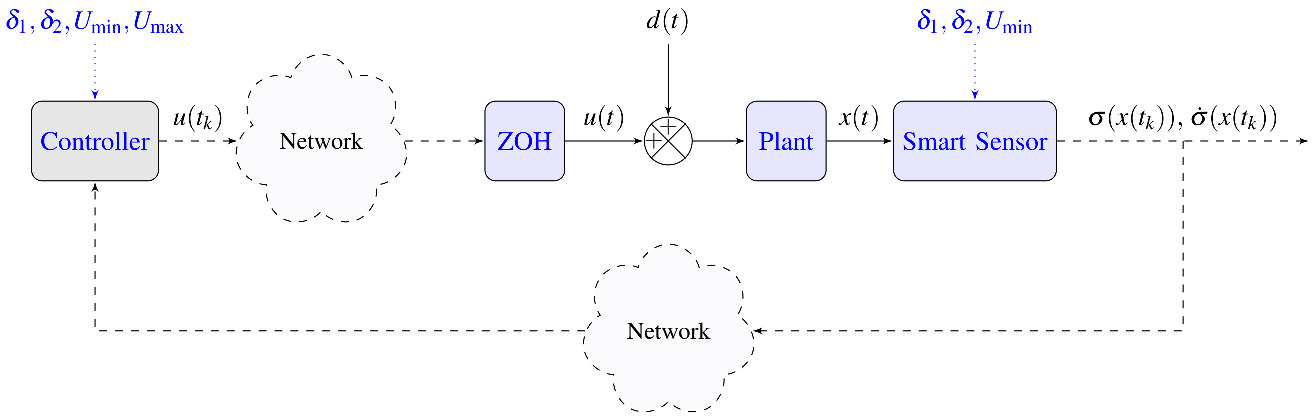

Fig. 1. The proposed Event-Triggered Second Order Sliding Mode control scheme.

while $a(x(t)) : \Omega \to \mathbb{R}^n$ and $b(x(t)) : \Omega \to \mathbb{R}^n$ are uncertain functions of class C^1. Moreover, system (1) is affected by the external disturbance $d_{\mathrm{m}}(\cdot)$ assumed to be matched, i.e.,

$$d_{\mathrm{m}}(x(t),t) = b(x(t))d(t) \tag{2}$$

such that

$$d \in \mathcal{D} \subset \mathbb{R} \tag{3}$$

where $\mathcal{D}^{\sup} \triangleq \sup_{d\in\mathcal{D}}\{|d|\}$ is a known positive constant.

Define a suitable output function $\sigma(x(t)) : \Omega \to \mathbb{R}$ sufficiently smooth and of class C^2. This function is called "sliding variable" and it has to be such that: if $u(t)$ in (1) is designed so that, in a finite time $t_{\mathrm{r}}^\star \geq t_0$ (ideal reaching time), $\sigma(x(t_{\mathrm{r}}^\star)) = 0\ \forall x_0 \in \Omega$ and $\sigma(x(t)) = 0\ \forall t > t_{\mathrm{r}}^\star$, then $\forall t \geq t_{\mathrm{r}}^\star$ the origin is an asymptotically stable equilibrium point of (1) constrained to $\sigma(x(t)) = 0$. Then, the input-output map is

$$\begin{cases} \dot{x}(t) &= a(x(t)) + b(x(t))(u(t) + d(t)) \\ y(t) &= \sigma(x(t)) \\ x(t_0) = x_0 \end{cases} \tag{4}$$

Assume that (4) is complete in Ω, i.e., $x(t) \in \Omega$ and, for each initial state x_0 and each control u, $x(t)$ is defined $\forall\, t \geq t_0$. Moreover, assume that (4) has a uniform relative degree equal to 2 and admits a global normal form in Ω, i.e., there exists a global diffeomorphism of the form $\Phi(x) : \Omega \to \Phi_\Omega \subset \mathbb{R}^n$

$$\Phi(x) = \begin{pmatrix} \Psi(x) \\ \sigma(x) \\ a(x)\cdot\nabla\sigma(x) \end{pmatrix} = \begin{pmatrix} x_{\mathrm{r}} \\ \xi \end{pmatrix}$$

$$\Psi : \Omega \to \mathbb{R}^{n-2}, \quad x_{\mathrm{r}} \in \mathbb{R}^{n-2}, \quad \xi = \begin{pmatrix} \sigma(x) \\ \dot{\sigma}(x) \end{pmatrix} \in \mathbb{R}^2$$

such that,

$$\begin{cases} \dot{x}_{\mathrm{r}} &= a_{\mathrm{r}}(x_{\mathrm{r}}, \xi) & \text{(5a)} \\ \dot{\xi}_1 &= \xi_2 & \text{(5b)} \\ \dot{\xi}_2 &= f(x_{\mathrm{r}}, \xi) + g(x_{\mathrm{r}}, \xi)(u + d) & \text{(5c)} \\ y &= \xi_1 & \text{(5d)} \\ \xi(t_0) = \xi_0 & & \text{(5e)} \end{cases}$$

with

$$a_{\mathrm{r}}(x_{\mathrm{r}}, \xi) = \frac{d\Psi}{dx}(\Phi^{-1}(x_{\mathrm{r}}, \xi))\, a(\Phi^{-1}(x_{\mathrm{r}}, \xi))$$
$$f(x_{\mathrm{r}}, \xi) = a(\Phi^{-1}(x_{\mathrm{r}}, \xi)) \cdot \nabla(a(\Phi^{-1}(x_{\mathrm{r}}, \xi)) \cdot \nabla\sigma(\Phi^{-1}(x_{\mathrm{r}}, \xi)))$$
$$g(x_{\mathrm{r}}, \xi) = b(\Phi^{-1}(x_{\mathrm{r}}, \xi)) \cdot \nabla(a(\Phi^{-1}(x_{\mathrm{r}}, \xi)) \cdot \nabla\sigma(\Phi^{-1}(x_{\mathrm{r}}, \xi)))$$

Note that, by the assumption of uniform relative degree, it yields

$$g(x_{\mathrm{r}}, \xi) \neq 0, \quad \forall (x_{\mathrm{r}}, \xi) \in \Phi_\Omega \tag{6}$$

In the literature, subsystem (5b)-(5e) is called "auxiliary system" [6]. Since $a_{\mathrm{r}}(\cdot)$, $f(\cdot)$, $g(\cdot)$ are continuous functions and Φ_Ω is a bounded set, one has

$$\exists F > 0 \quad : |f(x_{\mathrm{r}}, \xi)| \leq F \quad \forall (x_{\mathrm{r}}, \xi) \in \Phi_\Omega \tag{7}$$
$$\exists G_{\max} > 0 \quad : \quad g(x_{\mathrm{r}}, \xi) \leq G_{\max} \quad \forall (x_{\mathrm{r}}, \xi) \in \Phi_\Omega \tag{8}$$

Moreover, one can also assume that

$$\exists G_{\min} > 0 \quad : \quad g(x_{\mathrm{r}}, \xi) \geq G_{\min} \quad \forall (x_{\mathrm{r}}, \xi) \in \Phi_\Omega \tag{9}$$

Relying on (5)-(9), we can introduce a preliminary control problem: *design a feedback control law*

$$u(t) = \kappa(\sigma(x(t)), \dot{\sigma}(x(t))) \tag{10}$$

such that

$$\forall x_0 \in \Omega,\ \exists t_{\mathrm{r}}^\star \geq t_0 :\ \sigma(x(t)) = \dot{\sigma}(x(t)) = 0,\ \forall t \geq t_{\mathrm{r}}^\star \tag{11}$$

in spite of the uncertainties.

Remark 1: Note that if the preliminary control problem is solved, because of the choice of σ, one has that $\forall x_0 \in\ \Omega$, the origin of the state space is a robust asymptotically stable equilibrium point for (1)-(9).

Consider now how the control law (10) is realized in field implementations. Typically, the state is sampled at time instants $t_k, k \in \mathbb{N}$, and the control law is held constant between two successive samplings by using a zero-order-hold (ZOH). In conventional implementation, the sequence $\{t_k\}_{k\in\mathbb{N}}$ is periodic and the control approach is classified as "time-triggered".

In the present paper, we want to design a solution for NCSs, therefore reducing the transmissions over the network.

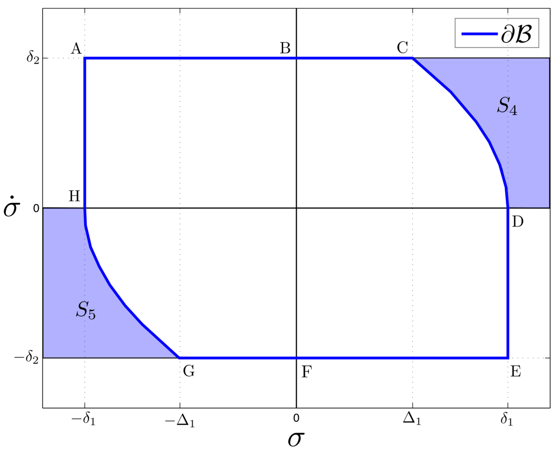

Fig. 2. Representation of regions S_4 and S_5 for the triggering condition.

Then, the idea is to introduce a triggering condition, so that the values of σ and $\dot{\sigma}$ are transmitted only when such a condition is satisfied. As a consequence, the control law is computed and sent to the plant only at the triggering time instants, and the overall control strategy is of "event-triggered" type.

Taking into account the previous considerations, and making reference to (5)-(9), we can move from the preliminary control problem to the formulation of the problem which will be actually solved: *design a feedback control law*

$$u(t) = u(t_k) = \kappa(\sigma(x(t_k)), \dot{\sigma}(x(t_k))) \tag{12}$$

$\forall t \in [t_k, t_{k+1}[,\ t_k, t_{k+1} \in \mathcal{T},\ k \in \mathbb{N}$, *$\mathcal{T}$ being the set of the triggering time instants, such that*

$$\forall x_0 \in \Omega, \exists t_\mathrm{r} \geq t_0 : |\sigma(x(t))| \leq \delta_1, |\dot{\sigma}(x(t))| \leq \delta_2, \forall t \geq t_\mathrm{r} \tag{13}$$

with δ_1 and δ_2 positive constants arbitrarily set.

Remark 2: Note that, when the natural relative degree of system (1) is equal to 1, the foregoing problem can be analogously formulated by artificially increasing the relative degree of the system. The motivation for solving that problem in presence of a system having unitary relative degree could be the so-called "chattering alleviation" [6], [19]

III. The New Proposal: Event-Triggered Second Order Sliding Mode Control

Consider the ET-SOSM control scheme reported in Fig. 1. It contains two key blocks: the smart sensor and the SOSM controller. These blocks are hereafter detailed.

A. The Smart Sensor

We assume that the considered sensor is smart in the sense that it has some computation capability, i.e., it is able to verify a triggering condition. The triggering condition adopted in this paper is the following

$$(\sigma, \dot{\sigma}) \in \{S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5\} \tag{14}$$

where

$$\begin{aligned}
S_1 &\triangleq \{(\sigma, \dot{\sigma}) : \ |\dot{\sigma}| \geq \delta_2\} \\
S_2 &\triangleq \{(\sigma, \dot{\sigma}) : \ \sigma \geq \delta_1,\ -\delta_2 < \dot{\sigma} \leq 0\} \\
S_3 &\triangleq \{(\sigma, \dot{\sigma}) : \ \sigma \leq -\delta_1,\ 0 \leq \dot{\sigma} < \delta_2\} \\
S_4 &\triangleq \left\{(\sigma, \dot{\sigma}) : \ \sigma \geq -\frac{\dot{\sigma}|\dot{\sigma}|}{2U_{\min}} + \delta_1,\ 0 < \dot{\sigma} < \delta_2\right\} \\
S_5 &\triangleq \left\{(\sigma, \dot{\sigma}) : \ \sigma \leq -\frac{\dot{\sigma}|\dot{\sigma}|}{2U_{\min}} - \delta_1,\ -\delta_2 < \dot{\sigma} < 0\right\}
\end{aligned}$$

with δ_1 and δ_2 positive constants arbitrarily set. The regions S_4 and S_5 are graphically represented in Fig. 2. The control amplitude $U_{\min} < U_{\max}$ is able to dominate only the drift term $f(x_\mathrm{r}, \xi)$ in (5). Then, by virtue of bounds (7) and (9), it is necessary to impose that

$$U_{\min} > \frac{F}{G_{\min}} \tag{15}$$

Only when the triggering condition (14) is true, are σ and $\dot{\sigma}$ transmitted by the sensor over the network, so that the control law is computed and sent to the plant.

B. The Second Order Sliding Mode Controller

Before introducing the proposed control algorithm, let us define the convergence set

$$\mathcal{B} \triangleq \mathbb{R}^2 \setminus \{S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5\} \tag{16}$$

In the following sections $\partial\mathcal{B}$ will denote the boundary of $\mathcal{B}$, and σ_k, $\dot{\sigma}_k$ will denote the values of the sliding variable and its first time derivative at the triggering time instants t_k, $k \in \mathbb{N}$.

To solve the control problem (12), (13) formulated in Section II, we design an event-triggered SOSM control scheme which uses two different control laws.

Control Law 1:
Let $\sigma(x(t_0)) = \sigma_0$ and $\dot{\sigma}(x(t_0)) = \dot{\sigma}_0$ be the initial conditions of the sliding variable and its first time derivative, respectively. If $(\sigma_0, \dot{\sigma}_0) \notin \{\mathcal{B} \cup \partial\mathcal{B}\}$, the control law (12) can be chosen as the classical bang-bang time-optimal control law depending on both σ_k and $\dot{\sigma}_k$, i.e.,

$$u(t_k) = \begin{cases} -U_{\max} & \text{if} \quad \left\{\sigma_k > -\frac{\dot{\sigma}_k|\dot{\sigma}_k|}{2U_{\max}}\right\} \cup \\ & \qquad \left\{\sigma_k = -\frac{\dot{\sigma}_k|\dot{\sigma}_k|}{2U_{\max}} \cap \sigma_k < 0\right\} \\ +U_{\max} & \text{if} \quad \left\{\sigma_k < -\frac{\dot{\sigma}_k|\dot{\sigma}_k|}{2U_{\max}}\right\} \cup \\ & \qquad \left\{\sigma_k = -\frac{\dot{\sigma}_k|\dot{\sigma}_k|}{2U_{\max}} \cap \sigma_k > 0\right\} \end{cases} \tag{17}$$

where $U_{\max} > U_{\min}$ is a positive value suitably selected in order to enforce a sliding mode, even in presence of the external disturbance d_m. This control law is applied only during the reaching phase, i.e., till $(\sigma, \dot{\sigma})$ reaches the boundary $\partial\mathcal{B}$. From the reaching time instant t_r onwards, the Control Law 2 will be applied.

Control Law 2:
If $(\sigma_0, \dot{\sigma}_0) \in \{\mathcal{B} \cup \partial\mathcal{B}\}$, or after the reaching phase (if

$(\sigma_0, \dot{\sigma}_0) \notin \{\mathcal{B} \cup \partial\mathcal{B}\}$), the Control Law 1 is modified as follows

$$u(t_k) = \begin{cases} -U(t_k) & \text{if} \quad \{\dot{\sigma}_k > \delta_2\} \bigcup \\ & \quad\quad \{\dot{\sigma}_k > 0 \cap \sigma_k = 0\} \bigcup \\ & \quad\quad \{|\dot{\sigma}_k| \le \delta_2 \cap \sigma_k > 0\} \\ \\ +U(t_k) & \text{if} \quad \{\dot{\sigma}_k < -\delta_2\} \bigcup \\ & \quad\quad \{\dot{\sigma}_k < 0 \cap \sigma_k = 0\} \bigcup \\ & \quad\quad \{|\dot{\sigma}_k| \le \delta_2 \cap \sigma_k < 0\} \end{cases} \tag{18}$$

where the control amplitude $U(t_k)$, $k \in \mathbb{N}$, can assume two different values, i.e.,

$$U(t_k) = \begin{cases} U_{\min} & \text{if} \quad \Big\{0 < \dot{\sigma}_k \le \delta_2 \cap \\ & \quad\quad -\delta_1 \le \sigma_k < \Delta_1\Big\} \bigcup \\ & \quad\quad \Big\{-\delta_2 \le \dot{\sigma}_k < 0 \cap \\ & \quad\quad -\Delta_1 < \sigma_k \le \delta_1\Big\} \\ \\ U_{\max} & \quad \text{otherwise} \end{cases} \tag{19}$$

with

$$\Delta_1 \triangleq -\frac{\delta_2^2}{2U_{\min}} + \delta_1 \tag{20}$$

indicated in Fig. 2.

The control law (12), with $u(t_k)$ as in (17)-(19), and the triggering condition (14) give rise to the Event-Triggered SOSM (ET-SOSM) control strategy that we propose to steer $\sigma, \dot{\sigma}$ to the convergence set $\mathcal{B}$.

IV. Stability Analysis

In this section, the properties of the proposed ET-SOSM control strategy are analyzed. For the readers' convenience, we introduce the following definitions:

Definition 1: The sliding variable σ and its first time derivative $\dot{\sigma}$ are said to be ultimately bounded with respect to the set B_{δ_i}, $i = 1, 2$, respectively, if in a finite time t_{r_i}, $i = 1, 2$, they enter the bounded set B_{δ_i}, $i = 1, 2$, and there remain for all subsequent time instants, i.e.,

$$\begin{aligned} &\forall x_0 \in \Omega,\ \exists\, t_{\mathrm{r}_1} \ge t_0 : \sigma(x(t)) \in B_{\delta_1} \quad \forall t \ge t_{\mathrm{r}_1} \\ &\forall x_0 \in \Omega,\ \exists\, t_{\mathrm{r}_2} \ge t_0 : \dot{\sigma}(x(t)) \in B_{\delta_2} \quad \forall t \ge t_{\mathrm{r}_2} \end{aligned}$$

where

$$\begin{aligned} B_{\delta_1} &\triangleq \left\{\sigma(x(t)) :\ |\sigma(x(t))| \le \delta_1\right\} \\ B_{\delta_2} &\triangleq \left\{\dot{\sigma}(x(t)) :\ |\dot{\sigma}(x(t))| \le \delta_2\right\} \end{aligned}$$

with δ_1, δ_2 positive constants.

Definition 2: The solution $(\sigma, \dot{\sigma})$ to the auxiliary system (5b)-(5e) is said to be ultimately bounded with respect to the closed set $\mathcal{B}$ if in a finite time $t_{\mathrm{r}} = \max(t_{\mathrm{r}_1}, t_{\mathrm{r}_2})$ it enters the closed set $\mathcal{B}$ and there remains for all subsequent time instants, i.e.,

$$\forall x_0 \in \Omega,\ \exists\, t_{\mathrm{r}} \ge t_0 : (\sigma(x(t)), \dot{\sigma}(x(t))) \in \{\mathcal{B} \cup \partial\mathcal{B}\} \quad \forall t \ge t_{\mathrm{r}}$$

Definition 3: Let $(\sigma, \dot{\sigma})$ be the solution to the auxiliary system (5b)-(5e) starting from the initial condition $(\sigma_0, \dot{\sigma}_0)$. The bounded set B_{δ_1} is said to be positively invariant if $(\sigma_0, \dot{\sigma}_0) \in \{\mathcal{B} \cup \partial\mathcal{B}\}$ implies that $\sigma(x(t)) \in B_{\delta_1}\ \forall t \ge t_0$.

Now, the following results can be proved, but, because of space limitation, the corresponding proofs are omitted.

Lemma 1: Given the auxiliary system (5b)-(5e) starting from the initial condition $(\sigma_0, \dot{\sigma}_0) \notin \{\mathcal{B} \cup \partial\mathcal{B}\}$ with δ_1, δ_2 arbitrarily set, controlled via (12), (17) and the triggering condition (14), then, the solution $(\sigma, \dot{\sigma})$ to (5b)-(5e) is steered to the set $\{\mathcal{B} \cup \partial\mathcal{B}\}$ in a finite time.

Remark 3: Note that during the reaching of the set $\{\mathcal{B} \cup \partial\mathcal{B}\}$, by virtue of Lemma 1, it is possible to update the control law only when the auxiliary state-space trajectory crosses the so-called "switching line", i.e., when the following condition holds

$$\sigma = -\frac{\dot{\sigma}|\dot{\sigma}|}{2U_{\max}}$$

Lemma 2: Given the auxiliary system (5b)-(5e) starting from the initial condition $(\sigma_0, \dot{\sigma}_0) \in \{\mathcal{B} \cup \partial\mathcal{B}\}$ with δ_1, δ_2 arbitrarily set, controlled via (12), (18)-(19) and the triggering condition (14), then, if the maximum control amplitude $U_{\max}$ in (19) is such that

$$U_{\max} > \frac{F + U_{\min}}{G_{\min}} + \mathcal{D}^{\sup} \tag{21}$$

the boundary layer B_{δ_1} is a positively invariant set for σ and, at most, $\dot{\sigma}$ switches along the bounds $\pm\delta_2$.

Now, relying on Lemma 1 and Lemma 2, one can prove the major result concerning the evolution of the auxiliary system (5b)-(5e) controlled via the proposed ET-SOSM control strategy.

Theorem 1: Given the auxiliary system (5b)-(5e) starting from the initial condition $(\sigma_0, \dot{\sigma}_0) \notin \{\mathcal{B} \cup \partial\mathcal{B}\}$ with δ_1, δ_2 arbitrarily set, controlled via (12), (17)-(19) with the additional constraint (21) and the triggering condition (14), then, the solution $(\sigma, \dot{\sigma})$ to (5b)-(5e) is ultimately bounded with respect to the closed set $\mathcal{B}$ and, at most, $\dot{\sigma}(t)$ switches along the bounds $\pm\delta_2$.

Remark 4: Note that the proposed control scheme, because of its event-triggered nature, cannot generate an ideal sliding mode, but only a "practical sliding" mode. However, by virtue of the Regularization Theorem in book [4, Chapter 2], it can be proved that also the state of system (1) is ultimately bounded. This implies that the control problem formulated in Section II is equivalent to the problem of designing a bounded control capable of enforcing the "practical stability" of (1) with respect to $(t_0, t_{\mathrm{p}}, \Omega, \Omega_{\mathrm{p}}, \mathcal{D})$, where $\Omega_{\mathrm{p}} \subset \Omega$ and $t_{\mathrm{p}} \ge t_{\mathrm{r}}$, that is, according to LaSalle and Lefshetz [20], $\forall t_0 \ge 0, \forall x_0 \in \Omega, \forall d \in \mathcal{D}$, one has that $x(t) \in \Omega_{\mathrm{p}}, \forall t \ge t_{\mathrm{p}}$.

V. Simulations

In this section, in order to illustrate the properties of the proposed ET-SOSM control strategy, an academic example is briefly discussed. Consider a perturbed chain of 3 integrators,

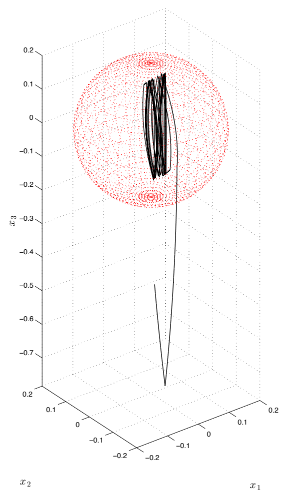

Fig. 3. Trajectory of the controlled system state.

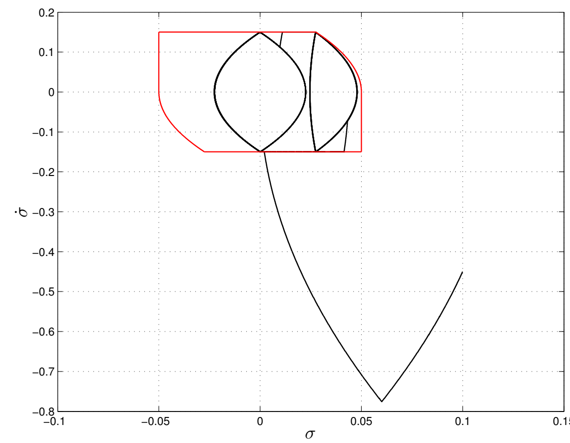

Fig. 4. Trajectory of the auxiliary system state with the convergence set.

i.e.,

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = x_3(t) \\ \dot{x}_3(t) = 0.5x_2(t) - x_3(t) + u(t) + d(t) \\ y(t) = x_1(t) + x_2(t) \end{cases} \tag{22}$$

where the external disturbance $d(t)$, acting from $t = 10.5\,\mathrm{s}$ to $t = 16\,\mathrm{s}$, is such that $\mathcal{D}^{\sup} = 4.4$. Let t_0 be equal to 0, and the initial condition be $x_0 = [0.05\ 0.05\ \ -0.5]^T$. Then, the system is stabilized by choosing the sliding variable $\sigma(x(t))$ as the controlled variable $y(t)$, such that the corresponding auxiliary system results in being

$$\begin{cases} \dot{\xi}_1(t) = \xi_2(t) \\ \dot{\xi}_2(t) = 0.5x_2(t) + u(t) + d(t) \end{cases} \tag{23}$$

with the initial condition $\xi_0 = [0.1\ \ -0.45]^T$. The ET-SOSM control parameters are $U_{\min} = 0.5$, $U_{\max} = 5.0$, $\delta_1 = 0.05$ and $\delta_2 = 0.15$. The trajectory of the state of the controlled system is illustrated in Fig. 3. As expected, the state of system (22) is steered to a small vicinity of the origin in a finite time and there remains in spite of the action of the external disturbance d. The auxiliary state-space trajectory with the convergence set $\mathcal{B}$ is illustrated in Fig. 4, showing that the trajectory of the auxiliary system (23) is steered to the convergence set $\mathcal{B}$ in a finite time. More specifically, Fig. 5 shows that the sliding variable σ is steered to the boundary layer B_{δ_1} in a finite time $t_{r_1} = 0.08\,\mathrm{s}$, and it remains inside for all subsequent time instants. Moreover, Fig. 6 shows that also $\dot{\sigma}$ is steered to the boundary layer B_{δ_2} in a finite time $t_{r_2} = 0.19\,\mathrm{s}$, and it remains inside for any subsequent time instant or, at worst, it switches along the boundary of B_{δ_2}. Then, one can conclude that the auxiliary state-space trajectory is ultimately bounded in $\mathcal{B}$. The time evolution of the control variable $u(t)$ is shown in Fig. 7, while the flag function f_t, representing the numbers of triggering events, i.e., the transmission of the actual state of the auxiliary system, is illustrated in Fig. 8. More specifically, f_t is a flag equal to 1 when the triggering condition (14) holds. Note that, since the initial condition $(\sigma_0, \dot{\sigma}_0)$ is known, only one state transmission occurs during the reaching phase (see zoom in Fig. 8), i.e., when the auxiliary state-space trajectory crosses the switching line, while the number of triggering events increases when the disturbance d acts. However, considering a sampling time $T_s = 1 \times 10^{-4}\,\mathrm{s}$, and a simulation horizon $T = 20\,\mathrm{s}$, the number of state transmissions with the proposed ET-SOSM control algorithm is 1764, i.e., 99.1 % less then the number required by the conventional time-driven implementation. Note that reducing the parameters δ_1 and δ_2 results in improving the convergence accuracy.

VI. Conclusions

In this paper a novel Second Order Sliding Mode control strategy of event-triggered type for a class of nonlinear uncertain systems is presented. The proposed control scheme requires the transmission of the state of the controlled auxiliary system only when a suitably defined triggering condition is verified. In spite of this, it guarantees satisfactory stability properties of the controlled system. In particular, in the paper we prove that the solution of the controlled auxiliary system is ultimately bounded in a prescribed convergence set where the approximability property of classical Sliding Mode control holds. We also observe that this implies the practical stability of the considered uncertain nonlinear system. These results are attained in spite of the significant reduction of the number of transmissions of the system state. Simulation assessment confirms the theoretical results.

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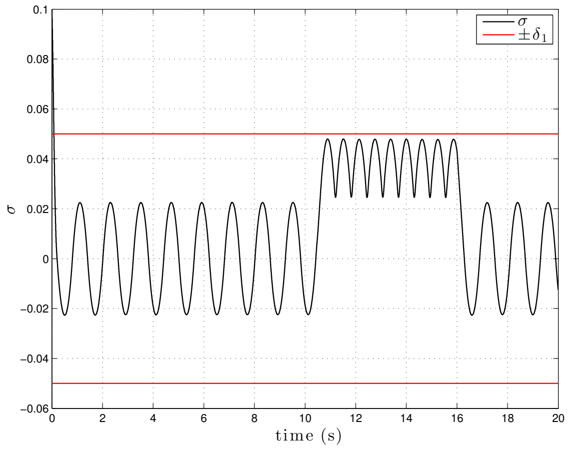

Fig. 5. Time evolution of σ with the boundary layer B_{δ_1}.

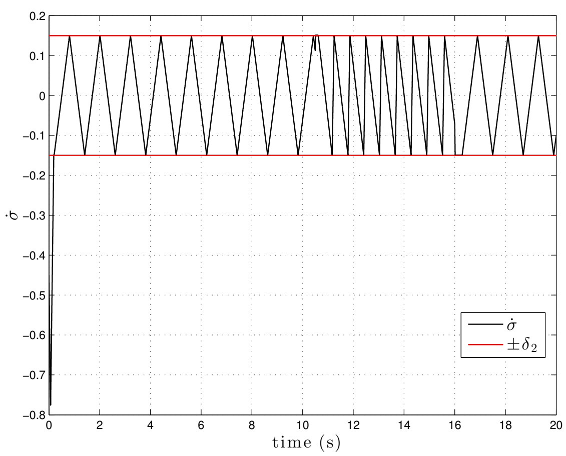

Fig. 6. Time evolution of $\dot{\sigma}$ with the boundary layer B_{δ_2} .

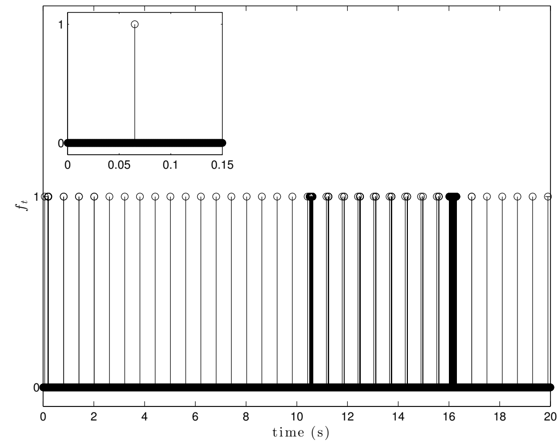

Fig. 7. Time evolution of the control variable u fed into the plant.

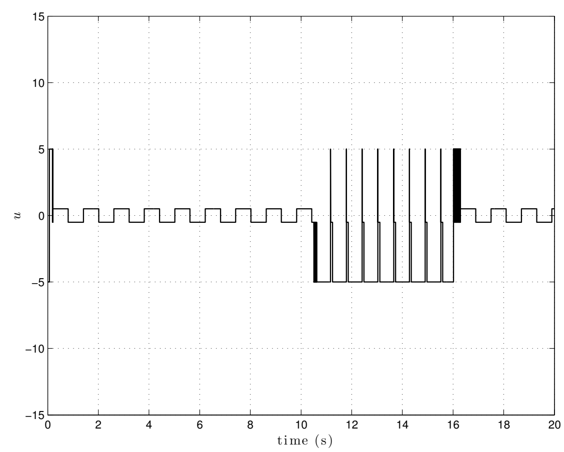

Fig. 8. Time instants when transmissions occur, i.e., $f_t = 1$.

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