

Distributed Second Order Sliding Modes for Optimal Load Frequency Control★

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Abstract—This paper proposes a Distributed Second Order Sliding Mode (D-SOSM) control strategy for Optimal Load Frequency Control (OLFC) in power networks, where besides frequency regulation also minimization of generation costs is achieved. Because of unknown load dynamics and possible network parameters uncertainties, the sliding mode control methodology is particularly appropriate for the considered control problem. This paper considers a power network partitioned into control areas, where each area is modelled by an equivalent generator including second-order turbine-governor dynamics. On a suitable designed sliding manifold, the controlled system exhibits an incremental passivity property that allows us to infer convergence to a zero steady state frequency deviation minimizing the generation costs.

I. INTRODUCTION

As a result of power mismatch between generation and demand, the frequency in the power system can deviate from its nominal value. Regulating the frequency by Load Frequency Control (LFC) in power systems composed of interconnected Control Areas (CAs) is a challenging issue and it is unsure if current implementations are adequate to deal with an increasing share of renewable energy sources [1].

Traditionally, the LFC is performed at each CA by a primary droop control and a secondary Proportional-Integral (PI) control. To cope with the increasing uncertainties affecting a CA and to improve the controllers performance, advanced control techniques have been proposed to redesign the conventional LFC schemes, such as Model Predictive Control (MPC) [2], adaptive control [3], fuzzy control [4] and Sliding Mode (SM) control. However, due to the predefined power flows through the tie-lines, the possibility of achieving economically optimal LFC is lost [5]. Besides improving the stability and the dynamic performance of power systems, new control strategies are additionally required to reduce the operational costs of LFC [6]. In this paper we propose a novel distributed *Optimal* LFC (OLFC) scheme that incorporates the economic

dispatch into the LFC, departing from the conventional tie-line requirements.

In order to obtain OLFC, the vast majority of solutions appearing in the literature fit in one of two categories. First, the economic dispatch problem is distributively solved by a primal-dual algorithm converging to the solution of the associated Lagrangian dual problem [7]–[9]. This approach generally requires measurements of the loads or the power flows, which is undesirable in a LFC scheme. This issue is avoided by the second class of solutions, where a distributed consensus algorithm is employed to converge to a state of identical marginal costs, solving the economic dispatch problem in the unconstrained case [10]–[13]. The proposed solution in this work fits in the second category, where we utilize a distributed sliding model control scheme to achieve consensus in the marginal costs.

Sliding mode control [14], [15] has been used to improve the conventional LFC schemes [16], possibly together with fuzzy logic [17] and disturbances observers [18]. However, the proposed use of SM to obtain a distributed OLFC scheme is new and can offer a few advantages over the previous results on OLFC. Foremost, it is possible to incorporate the widely used second-order model for the turbine-governor dynamics that is currently neglected in the analytical OLFC studies.

In this paper, we adopt a *nonlinear* model of a power network partitioned into control areas having an arbitrarily complex and meshed topology. The generation side is modelled by an equivalent generator including second-order turbine-governor dynamics, where the proposed control scheme continuously adjusts the governor set point. Conventional SM controllers can suffer from the notorious drawback known as chattering effect, due to the discontinuous control input. To alleviate this issue, we incorporate the well known Suboptimal Second Order Sliding Mode (SSOSM) control algorithm [19].

Relying on an incremental passivity property of the power network [10], [20], we design a suitable sliding manifold, such that, when the controlled system is constrained to this manifold, the frequency deviation asymptotically converges to zero and the total generation costs are minimized. This result is obtained by avoiding the measurement of the power demand and the use of observers [21], which is an element concurring to the ease of practical implementation of the proposed control strategy.

II. NETWORK MODEL

In this section the dynamic model of a power grid partitioned into control areas is presented. The dynamic behaviour

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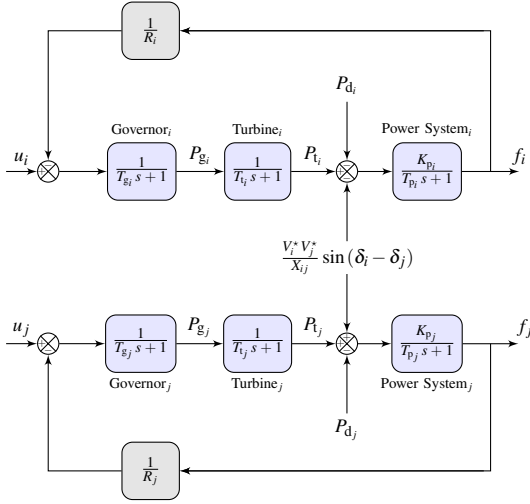


Fig. 1. Block diagram of two interconnected control areas.

of a single control area is described by an equivalent thermal power plant with a non-reheat turbine, which is commonly represented by second order turbine-governor dynamics.

Consider a power network consisting of n interconnected control areas. The network topology is represented by a connected and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the nodes $\mathcal{V} = \{1, \dots, n\}$, represent the control areas and the edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V} = \{1, \dots, m\}$, represent the transmission lines connecting the areas. The topology can be described by its corresponding incidence matrix $D \in \mathbb{R}^{n \times m}$. Then, by arbitrary labeling with a '+' and a '-' the ends of edge k , one has that

$$D_{ik} = \begin{cases} +1 & \text{if } i \text{ is the positive end of } k \\ -1 & \text{if } i \text{ is the negative end of } k \\ 0 & \text{otherwise.} \end{cases}$$

Now, not distinguishing between generator and load buses, the governing dynamic equations of the i -th node are the following:

$$\begin{aligned} \dot{\delta}_i &= 2\pi f_i \\ \dot{f}_i &= -\frac{1}{T_{p_i}} f_i + \frac{K_{p_i}}{T_{p_i}} P_i - \frac{K_{p_i}}{T_{p_i}} P_{d_i} \\ &\quad - \frac{K_{p_i}}{T_{p_i}} \sum_{j \in \mathcal{N}_i} \frac{V_i^* V_j^*}{X_{ij}} \sin(\delta_i - \delta_j), \end{aligned} \quad (1)$$

where \mathcal{N}_i is the set of nodes (i.e., control areas) connected to the i -th node by transmission lines. Note that we have assumed that the network is lossless, which is generally the case in high voltage transmission networks. Moreover, P_i in (1) is the power generated by the i -th thermal plant, and it can be expressed as the output of the following second order dynamic system that describes the behaviour of both the governor and the turbine of the thermal power plant, i.e.,

$$\begin{aligned} \dot{P}_i &= -\frac{1}{T_i} P_i + \frac{1}{T_i} P_{g_i} \\ \dot{P}_{g_i} &= -\frac{1}{R_i T_{g_i}} f_i - \frac{1}{T_{g_i}} P_{g_i} + \frac{1}{T_{g_i}} u_i. \end{aligned} \quad (2)$$

The main symbols used in systems (1) and (2) are described in Table I, and a block diagram of the considered system

TABLE I
DESCRIPTION OF THE USED SYMBOLS

Symbol	Description
δ_i	Voltage angle variation
f_i	Frequency deviation
P_i	Turbine output power variation
P_{g_i}	Governor output variation
T_{p_i}	Time constant of the control area
T_i	Time constant of the turbine
T_{g_i}	Time constant of the governor
K_{p_i}	Gain of the control area
R_i	Speed regulation coefficient
V_i^*	Constant voltage
X_{ij}	Line reactance
u_i	Control input
P_{d_i}	Unknown power demand

with two control areas is represented in Fig. 1.

We now write system (1) and the turbine-governor dynamics in (2) compactly for all nodes $i \in \mathcal{V}$ as

$$\begin{aligned} \dot{\eta} &= 2\pi D^T f \\ \dot{f} &= -T_p^{-1} f + K_p T_p^{-1} P_t - K_p T_p^{-1} P_d \\ &\quad - K_p T_p^{-1} D \Gamma \text{Sin}(\eta), \end{aligned} \quad (3a)$$

$$\begin{aligned} \dot{P}_t &= -T_t^{-1} P_t + T_t^{-1} P_g \\ \dot{P}_g &= -R^{-1} T_g^{-1} f - T_g^{-1} P_g + T_g^{-1} u, \end{aligned} \quad (3b)$$

where $\eta = D^T \delta \in \mathbb{R}^m$, $f \in \mathbb{R}^n$, $P_t \in \mathbb{R}^n$, $P_g \in \mathbb{R}^n$, $\Gamma = \text{diag}\{\gamma_1, \dots, \gamma_m\}$, with $\gamma_k = V_i^* V_j^* / X_{ij}$, $\text{Sin}(\eta) = [\sin(\eta_1), \dots, \sin(\eta_m)]^T$, $P_d \in \mathbb{R}^n$ and $u \in \mathbb{R}^n$. Matrices T_p, T_t, T_g, K_p, R are suitable $n \times n$ diagonal matrices.

To permit the controller design, the following assumption is introduced.

Assumption 1 *The variables f_i, P_i, P_{g_i} are locally available at control area i . The unmatched disturbance P_{d_i} is unknown, constant and can be bounded as*

$$|P_{d_i}| \leq \mathcal{D}_i, \quad (4)$$

where \mathcal{D}_i is a positive constant available at control area i .

III. PROBLEM FORMULATION

Optimal LFC has two main objectives. First, the control scheme needs to regulate the frequency towards its nominal value, i.e.

$$\lim_{t \rightarrow \infty} f = \mathbf{0}. \quad (5)$$

Second, the OLFC should obtain an economic dispatch, i.e. it needs to minimize the total costs $C(P_t)$ of the power generation required to control the frequency

$$\begin{aligned} \min_{P_t} C(P_t) &= \min_{P_t} \sum_{i \in \mathcal{V}} C_i(P_i) \\ \text{s.t. } 0 &= \mathbf{1}_n^T P_t - \mathbf{1}_n^T P_d, \end{aligned} \quad (6)$$

where $\mathbf{1}_n \in \mathbb{R}^n$ is the vector containing all ones, while the equality constraint follows from the requirement of a zero frequency deviation at steady state. Before further elaborating on this, we make the assumption of existence of a steady state of the system under a constant control input \bar{u} .

Assumption 2 *Given a constant power demand P_d , there exist $\bar{u}, \bar{\eta} \in \mathcal{R}(D^T)$, $\bar{f} \in \mathcal{N}(D^T)$, $\bar{P}_t \in \mathbb{R}^n$ and $\bar{P}_g \in \mathbb{R}^n$ such that $(\bar{\eta}, \bar{f}, \bar{P}_t, \bar{P}_g)$ satisfies*

$$\begin{aligned} \mathbf{0} &= 2\pi D^T \bar{f} \\ \mathbf{0} &= -T_p^{-1} \bar{f} + K_p T_p^{-1} \bar{P}_t - K_p T_p^{-1} \bar{P}_d \\ &\quad - K_p T_p^{-1} D \Gamma \text{Sin}(\bar{\eta}), \end{aligned} \quad (7a)$$

$$\begin{aligned} \mathbf{0} &= -T_t^{-1} \bar{P}_t + T_t^{-1} \bar{P}_g \\ \mathbf{0} &= -R^{-1} T_g^{-1} \bar{f} - T_g^{-1} \bar{P}_g + T_g^{-1} \bar{u}. \end{aligned} \quad (7b)$$

From algebraic manipulations of (7) it follows that the steady state frequency deviation is given by

$$\bar{f} = \mathbf{1}_n \frac{\mathbf{1}_n^T (\bar{P}_t - P_d)}{\mathbf{1}_n^T K_p^{-1} \mathbf{1}_n}. \quad (8)$$

From (8) it becomes clear that we indeed require the equality constraint in (6) to have a zero frequency deviation at steady state. The generation costs associated to control area i are commonly described by a strictly convex linear-quadratic cost function

$$C_i(P_i) = \frac{1}{2} q_i P_i^2 + z_i P_i + s_i,$$

such that the total costs in the power network can be expressed as

$$C(P_t) = \frac{1}{2} P_t^T Q P_t + Z^T P_t + \mathbf{1}_n^T S, \quad (9)$$

where Q is a $n \times n$ positive definite diagonal matrix and $Z, S \in \mathbb{R}^n$. It is now possible to explicitly characterize the solution \bar{P}_t^{opt} to the optimization problem (6).

Lemma 1 *Given the cost function (9) with Q a positive definite diagonal matrix, the solution \bar{P}_t^{opt} to the optimization problem (6) satisfies*

$$\bar{P}_t^{\text{opt}} = Q^{-1} (\mathbf{1}_n \bar{\lambda} - Z), \quad (10)$$

with

$$\bar{\lambda} = \frac{\mathbf{1}_n^T P_d + \mathbf{1}_n^T Q^{-1} Z}{\mathbf{1}_n^T Q^{-1} \mathbf{1}_n} \in \mathbb{R}. \quad (11)$$

From (10) it follows that $Q \bar{P}_t^{\text{opt}} + Z = \mathbf{1}_n \bar{\lambda} \in \mathcal{R}(\mathbf{1}_n)$. Consequently, at the economic dispatch all the marginal costs associated to power generation are equal. However, note that in (11) the value of P_d is required, which is generally unavailable in practical cases. The proposed solution in the next section overcomes this issue by simultaneously solving (6) and controlling the frequency without load measurements.

Now we are in a position to formulate the control problem: *Let Assumptions 1 and 2 hold. Given system (3) and the optimization problem (6), design a distributed control scheme achieving frequency regulation and minimizing, at the steady state, the generation costs.*

IV. THE PROPOSED SOLUTION

In this section a Distributed Suboptimal Second Order Sliding Mode (D-SSOSM) control algorithm is proposed to solve the aforementioned control problem. To do so, the well established SSOSM controller proposed in [19] is applied to the power network augmented with a distributed control scheme proposed in [20], leading to an overall distributed solution.

In order to define (and converge to) a sliding manifold on which a useful passivity property of the turbine-governor can be established (see Lemmas 3 and 4), and to enforce optimality at steady state (see the proof of Theorem 1), we augment the state of system (3) with additional state variables $\vartheta_i, i = 1, \dots, n$. Their dynamics are given by

$$T_{\vartheta_i} \dot{\vartheta}_i = P_i - \vartheta_i - a_i \sum_{j \in \mathcal{N}_i^{\text{comm}}} (q_i \vartheta_i + z_i - (q_j \vartheta_j + z_j)), \quad (12)$$

where $\mathcal{N}_i^{\text{comm}}$ is the set of the nodes that communicate with node i , and a_i is a positive constant. Note that the induced communication is required to achieve optimality.

Remark 1 *The topology of the communication network is described by the Laplacian matrix L_c . The dynamics in (12) can now be expressed compactly for all nodes $i \in \mathcal{V}$ as*

$$T_{\vartheta} \dot{\vartheta} = P_t - \vartheta - A L_c (Q \vartheta + Z), \quad (13)$$

where $A \in \mathbb{R}^{n \times n}$ is a positive definite diagonal matrix suitably selected. A possible choice of A is provided in the next section.

To guarantee an optimal coordination throughout the whole network the following assumption is made:

Assumption 3 *The undirected graph corresponding to the topology of the communication network is connected.*

Consider now the power network (3) augmented with (13). We select the sliding variables vector $\sigma \in \mathbb{R}^n$ as

$$\sigma = M_1 f + M_2 P_t + M_3 P_g + M_4 \vartheta, \quad (14)$$

M_1, \dots, M_4 being constant $n \times n$ diagonal matrices suitable selected in order to assign the dynamics of the augmented system when $\sigma = 0$. The permitted values for M_1, \dots, M_4 follow from the stability analysis and should be chosen to enforce a useful passivity property of the turbine-governor on the corresponding sliding manifold. A further discussion is provided in Lemmas 3 and 4 in the next section.

Remark 2 *Because M_1, \dots, M_4 are diagonal matrices, each sliding variable σ_i is defined by only local variables at node i .*

We now continue by describing the controller that guarantees the convergence to the sliding manifold $\sigma = \dot{\sigma} = \mathbf{0}$. Since the system relative degree¹ is equal to 1, then, in order to obtain a continuous control input, the SSOSM control algorithm can be applied by artificially increasing the relative degree of the system. By defining the auxiliary variables vectors $\xi_1 = \sigma$ and $\xi_2 = \dot{\sigma}$, the so-called auxiliary system is

$$\begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = \varphi + gw \\ \dot{u} = w, \end{cases} \quad (15)$$

where ξ_2 is not measurable. Indeed, according to Assumption 1, P_d is unknown. Bearing in mind (14) and that $\dot{\sigma} = \varphi + gw$, it follows that $\varphi \in \mathbb{R}^n$ and $g \in \mathbb{R}^{n \times n}$ are given by

$$\begin{aligned} \varphi = & \left(M_1 T_p^{-2} + M_3 R^{-1} T_g^{-1} T_p^{-1} - M_2 T_t^{-1} R^{-1} T_g^{-1} \right. \\ & \left. + M_3 T_g^{-1} R^{-1} T_g^{-1} \right) f - \left(M_1 T_p^{-1} K_p T_p^{-1} \right. \\ & \left. + M_3 R^{-1} T_g^{-1} K_p T_p^{-1} + M_1 K_p T_p^{-1} T_t^{-1} - M_2 T_t^{-2} \right) P_t \\ & + \left(M_1 K_p T_p^{-1} T_t^{-1} - M_2 T_t^{-2} - M_2 T_t^{-1} T_g^{-1} \right. \\ & \left. + M_3 T_g^{-2} \right) P_g + \left(M_2 T_t^{-1} - M_3 T_g^{-1} \right) T_g^{-1} u \\ & + \left(M_1 T_p^{-1} + M_3 R^{-1} T_g^{-1} \right) \left(K_p T_p^{-1} P_d \right. \\ & \left. + K_p T_p^{-1} D \Gamma \text{Sin}(\eta) \right) \\ & - M_1 K_p T_p^{-1} D \Gamma \frac{d}{dt} \text{Sin}(\eta) + M_4 \ddot{\vartheta}, \\ g = & M_3 T_g^{-1}, \end{aligned} \quad (16)$$

with $\frac{d}{dt} \text{Sin}(\eta) = [\cos(\eta_1) \dot{\eta}_1, \dots, \cos(\eta_m) \dot{\eta}_m]^T$. Note that, φ, g are uncertain due to the presence of the unmeasurable power demand P_d and possible parameters uncertainties.

Remark 3 Note that the uncertain function φ in (16) depends on the system state and on the control input u . However, it is locally bounded since the operational region in practical cases is always bounded, and in a vicinity of the sliding manifold the control input u remains close to the so-called equivalent control [22].

Making reference to condition (4), and assuming that the parameters uncertainties are bounded, then φ and g can be bounded as

$$|\varphi_i| \leq \Phi_i, \quad i = 1, \dots, n \quad (17)$$

$$G_{\min_i} \leq gi \leq G_{\max_i}, \quad i = 1, \dots, n \quad (18)$$

Φ_i, G_{\min_i} and $G_{\max_i}, i = 1, \dots, n$, being positive constants. However, if the bounds Φ_i, G_{\min_i} and $G_{\max_i}, i = 1, \dots, n$, cannot be a-priori estimated, the adaptive version of the SSOSM algorithm proposed in [23] can be used in order to dominate the effect of the uncertainties.

¹The relative degree is the minimum order r of the time derivative $\sigma_i^{(r)}, i = 1, \dots, n$, of the sliding variable associated to the i -th node in which the control $u_i, i = 1, \dots, n$, explicitly appears.

To steer ξ_{1_i} and $\xi_{2_i}, i = 1, \dots, n$, to zero in a finite time even in presence of the uncertainties, the SSOSM algorithm [19] is used. Consequently, the control law for the i -th node is given by

$$w_i = -\alpha_i W_{\max_i} \text{sgn} \left(\xi_{1_i} - \frac{1}{2} \xi_{1, \max_i} \right), \quad (19)$$

with

$$W_{\max_i} > \max \left(\frac{\Phi_i}{\alpha_i^* G_{\min_i}}; \frac{4\Phi_i}{3G_{\min_i} - \alpha_i^* G_{\max_i}} \right), \quad (20)$$

$$\alpha_i^* \in (0, 1] \cap \left(0, \frac{3G_{\min_i}}{G_{\max_i}} \right). \quad (21)$$

In (19) the extremal values ξ_{1, \max_i} can be detected by implementing for instance a peak detection as in [22]. Moreover, note that the discontinuous SSOSM control law (19) only affects ξ_{2_i} , and the control u_i fed into the governor of the node i is continuous.

V. STABILITY ANALYSIS AND MAIN RESULT

In this section we study the stability of the proposed control scheme. Specifically, we prove that given the proposed control scheme, system (3) converges to the set where $f = \mathbf{0}$ and $P_t = \bar{P}_t^{\text{opt}}$. In order to invoke LaSalle's invariance principle later on, we make the following assumption on the differences of voltage angles at steady state, which is generally satisfied under normal operating conditions of the power network.

Assumption 4 At the steady state, $\bar{\eta} \in (-\frac{\pi}{2}, \frac{\pi}{2})^m$ holds.

Furthermore the analysis relies on the notion of incremental passivity [24], [25]. We now recall a useful result from [10]

Lemma 2 Let Assumptions 1 and 4 hold. System (3a) with input P_t and output f is an output strictly incrementally passive system, with respect to the steady state satisfying

$$\begin{aligned} \mathbf{0} &= 2\pi D^T \mathbf{0} \\ \mathbf{0} &= -T_p^{-1} \mathbf{0} + K_p T_p^{-1} \bar{P}_t^{\text{opt}} - K_p T_p^{-1} P_d \\ &\quad - K_p T_p^{-1} D \Gamma \text{Sin}(\bar{\eta}). \end{aligned} \quad (22)$$

Namely, there exists a storage function $U_1(f, \mathbf{0}, \eta, \bar{\eta})$ which satisfies the following incremental dissipation inequality

$$\dot{U}_1 = -f^T K_p^{-1} f + f^T (P_t - \bar{P}_t^{\text{opt}}). \quad (23)$$

In various studies on Optimal LFC, this passivity property has been exploited to derive suitable controllers in the absence of second-order turbine-governor dynamics. Unfortunately, the second order turbine-governor dynamics do not possess a useful passivity property that allows for a passive interconnection². To overcome this issue, the SSOSM control enforces the turbine-governor dynamics to converge in a finite time to a sliding manifold where this passivity property is recovered under the following assumption.

²This can be readily concluded from the observation that (3b) with input $-f$ and output P_t has relative degree 2.

Assumption 5 Let $M_1 > 0, M_2 \geq 0, M_3 > 0$ and $M_4 = -(M_2 + M_3)$ in (14). Furthermore, let $A = (M_2 + M_3)^{-1} M_1 Q$ in (13).

Note that this assumption can always be fulfilled. We first characterize this sliding manifold in the lemma below.

Lemma 3 Let Assumptions 2 and 5 hold. System (3b) augmented with (13) converges in a finite time t_r to the sliding manifold where

$$P_g = -M_3^{-1}(M_1 f + M_2 P_t + M_4 \vartheta), \quad \forall t \geq t_r. \quad (24)$$

The proof follows from applying the SSOSM controller (19)–(21) to each control area such that a second order sliding mode is enforced. As a result of Lemma 3 we can substitute (24) in (3), $\forall t \geq t_r$, obtaining the following reduced order system

$$\begin{aligned} \dot{\eta} &= 2\pi D^T f \\ T_p K_p^{-1} \dot{f} &= -K_p^{-1} f + P_t - P_d - D\Gamma \text{Sin}(\eta), \\ M_1^{-1} M_3 T_t \dot{P}_t &= -M_1^{-1}(M_2 + M_3)P_t - f - M_1^{-1} M_4 \vartheta \\ T_\vartheta \dot{\vartheta} &= P_t - \vartheta - AL_c(Q\vartheta + Z) \\ \sigma &= \mathbf{0}, \end{aligned} \quad (25b)$$

where the dynamics of the governor has been replaced by the equality constraint $\sigma = \mathbf{0}$. Indeed, one can observe that the dynamics of the governor can be obtained by differentiating (24). Incremental passivity of (25b) can now be proven.

Lemma 4 Let Assumptions 1, 2 and 5 hold. System (25b) with input $-f$ and output P_t is an incrementally passive system, with respect to the steady state satisfying

$$\begin{aligned} \mathbf{0} &= -M_1^{-1}(M_2 + M_3)\bar{P}_t^{\text{opt}} - \mathbf{0} - M_1^{-1} M_4 \bar{\vartheta} \\ \mathbf{0} &= \bar{P}_t^{\text{opt}} - \bar{\vartheta} - AL_c(Q\bar{\vartheta} + Z). \end{aligned} \quad (26)$$

Namely, the storage function

$$\begin{aligned} U_2 &= \frac{1}{2}(P_t - \bar{P}_t^{\text{opt}})^T M_1^{-1} M_3 T_t (P_t - \bar{P}_t^{\text{opt}}) \\ &+ \frac{1}{2}(\vartheta - \bar{\vartheta})^T M_1^{-1}(M_2 + M_3) T_\vartheta (\vartheta - \bar{\vartheta}), \end{aligned} \quad (27)$$

satisfies the following incremental dissipation inequality

$$\begin{aligned} \dot{U}_2 &= -(P_t - \vartheta)^T M_1^{-1}(M_2 + M_3)(P_t - \vartheta) \\ &- (Q\vartheta + Z)^T L_c(Q\vartheta + Z) - (P_t - \bar{P}_t^{\text{opt}})^T f, \end{aligned} \quad (28)$$

along the solutions to (25b).

Remark 4 Note that the term $-AL_c(Q\vartheta + Z)$ in (25b) is not needed to enforce the discussed passivity property, but is required to prove convergence to the economic efficient generation \bar{P}_t^{opt} . In fact, setting $A = 0$ still permits to infer frequency regulation in Theorem 1 below.

Now, we can prove the main result of this paper concerning the evolution of the augmented system controlled via the proposed D-SSOSM control strategy.

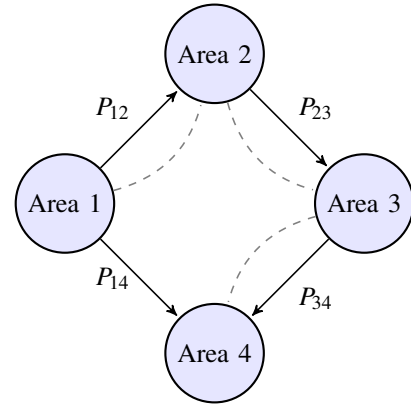


Fig. 2. Scheme of the considered power network partitioned into 4 control areas, where $P_{ij} = \frac{V_i^* V_j^*}{X_{ij}} \sin(\delta_i - \delta_j)$. The arrows indicate the positive direction of the power flows through the power network, while the dashed lines represent the communication network.

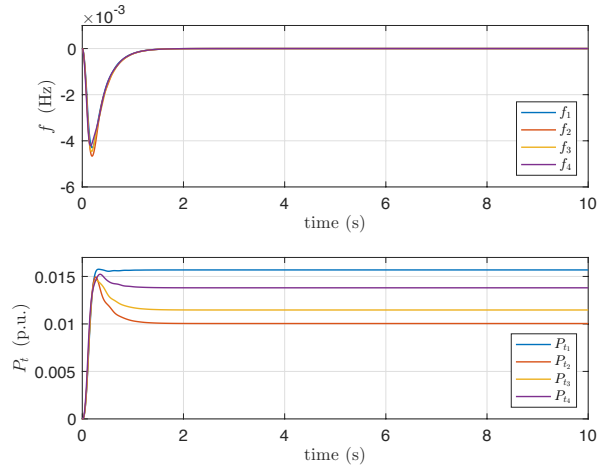


Fig. 3. Time evolution of the frequency deviation and generated power considering a power demand variation at the initial time instant $t_0 = 0$.

Theorem 1 Let Assumptions 1–5 hold. Consider system (3), augmented with the distributed averaging integrators (13) and controlled via (14)–(21). Then, the solutions of the closed-loop system starting in a neighbourhood of the equilibrium $(\bar{\eta}, \bar{f} = \mathbf{0}, \bar{P}_t^{\text{opt}}, \bar{\vartheta} = \bar{P}_t^{\text{opt}})$ approach the largest invariant set where $\bar{f} = \mathbf{0}$ and $P_t = \bar{P}_t^{\text{opt}}$.

The proof follows from evaluating the incremental storage function $U = U_1 + U_2$ along the solution to the reduced order system (25) and applying LaSalle’s invariance principle.

VI. SIMULATION RESULTS

In this section, the proposed control solution is assessed in simulation by implementing a power network partitioned into four control areas (e.g. the IEEE New England 39-bus system [26]). The topology of the power network is represented in Figure 2 together with the communication network (dashed lines). The line parameters are $\gamma_1 = 5.4$ p.u., $\gamma_2 = 5.0$ p.u., $\gamma_3 = 4.5$ p.u. and $\gamma_4 = 5.2$ p.u., while the network parameters and the power demand ΔP_{d_i} of each area

TABLE II
NETWORK PARAMETERS AND POWER DEMAND

	Area 1	Area 2	Area 3	Area 4
T_{p_i} (s)	21.0	25.0	23.0	22.0
T_{i_i} (s)	0.30	0.33	0.35	0.28
T_{g_i} (s)	0.080	0.072	0.070	0.081
K_{p_i} (Hz p.u. ⁻¹)	120.0	112.5	115.0	118.5
R_i (Hz p.u. ⁻¹)	2.5	2.7	2.6	2.8
T_{ϑ_i} (s)	0.33	0.33	0.33	0.33
q_i (10 ⁴ \$ h ⁻¹)	2.42	3.78	3.31	2.75
ΔP_{d_i} (p.u.)	0.010	0.015	0.012	0.014

are provided in Table II, where a base power of 1000 MW is assumed. The matrices in (14) are chosen as $M_1 = 3I_4$, $M_2 = I_4$, $M_3 = 0.1I_4$ and $M_4 = -(M_2 + M_3)$, $I_4 \in \mathbb{R}^{4 \times 4}$ being the identity matrix, while the control amplitude W_{\max_i} and the parameter α_i^* , $i = 1, \dots, 4$, in (19) are selected equal to 10 and 1, respectively. Note that, for the sake of simplicity, in the cost function (9) we select $Z = S = \mathbf{0}$. In simulation, the system is initially at the steady state, implying that all the sliding variables are equal to zero. Then, at the initial time instant $t_0 = 0$ s, the power demand in each area is increased according to the values reported in Table II. From Figure 3, one can observe that the frequency deviations converge asymptotically to zero after a transient where the frequency drops because of the increasing load. Indeed, one can note that the proposed controllers increase the power generation in order to reach again a zero steady state frequency deviation. Moreover, the total power demand is shared among the areas, minimizing the total generation costs. More precisely, by applying the proposed D-SSOSM, the total generation costs are 10 % less than the generation costs when each area would produce only for its own demand.

VII. CONCLUSIONS

A distributed suboptimal second order sliding mode control scheme is proposed to solve an optimal load frequency control problem in power systems affected by unmatched disturbances due to fluctuations in load demand. In the paper, we adopted the model of a power network partitioned into control areas, where each area is represented by an equivalent generator including second-order turbine-governor dynamics. Based on a suitable chosen sliding manifold the system, constrained to this manifold, possesses an incremental passivity property that is exploited to prove that the frequency deviation asymptotically converges to zero and economic optimality is achieved. An important feature of the proposed distributed control approach is that each controller does not require neither the measurement of the power demand nor load observers, increasing the practical applicability.

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