Distributed Optimal Load Frequency Control with Stochastic Wind Power Generation

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***Abstract*— Motivated by the inadequacy of conventional control methods for power networks with a large share of renewable generation, in this paper we study the (stochastic) passivity property of wind turbines based on the Doubly Fed Induction Generator (DFIG). Differently from the majority of the results in the literature, where renewable generation is ignored or assumed to be constant, we model wind power generation as a stochastic process, where wind speed is described by a class of stochastic differential equations. Then, we design a distributed control scheme that achieves load frequency control and economic dispatch, ensuring the stochastic stability of the controlled network.**

I. INTRODUCTION

The supply-demand balance is an essential control objective in power networks. Indeed, the supply-demand mismatch leads to frequency deviations from the nominal value, which eventually may result in stability disruptions [1], [2]. For this reason, the main control objective in power networks is the so-called Load Frequency Control (LFC). Additionally, another key objective is the minimization of the generation costs, also known as economic dispatch [3]. The economic dispatch together with the LFC is called in the literature *Optimal* LFC (OLFC) (see for instance [3]–[6] and the references therein). However, due to the growing share of renewable generation sources in power networks, the conventional control schemes may be not adequate [7].

Different control strategies achieving LFC and OLFC have been proposed for instance in [8]–[10] and [3], [6], [11], respectively (see also the references therein). However, in all these works, only conventional power generation is taken into account.

A. Motivation and Contributions

Nowadays, renewable generation sources are widespread in power networks, leading to an inevitable increase of uncertainties affecting the overall power system and its stability, resilience and reliability. For this reason, advanced control methods that guarantee the stability of the power system also in presence of time-varying renewable sources are necessary. Indeed, due to the random and unpredictable nature of some primary energy sources such as wind, the dynamic behaviour of renewables can be usually described by stochastic processes (*e.g.* Ito calculus), as shown for instance in [12], [13] for wind power generation. Also, [14] proposes wind speed models based on Stochastic Differential Equations (SDEs), which can be useful in wind turbine models. Differently from [3], [6], [8]–[11] and other relevant works on the topic, in this paper we couple the wind speed model introduced in [13] with the model of wind turbines based on the Doubly Fed Induction Generator (DFIG). Then, we present a distributed passivity-based control scheme achieving OLFC and ensuring the stochastic stability of the power network.

The main contributions of this paper can be summarized as follows: (i) the OLFC problem for nonlinear power networks including the turbine-governor model of conventional generators and the model of DFIG-based wind turbines is formulated, where the wind speed is modeled by an SDE; (ii) sufficient conditions for the stochastic passivity of the open-loop system are presented, facilitating the interconnection with passive control systems; (iii) a control scheme is proposed to obtain the passivity property of the DFIG-based wind turbine; (iv) the stochastic stability of the power network controlled by the distributed control scheme proposed in [3] is proved and OLFC objective is achieved.

B. Notation

The set of real numbers is denoted by $\mathbb{R}$. The set of positive (nonnegative) real numbers is denoted by $\mathbb{R}_{>0}$ ($\mathbb{R}_{\geq 0}$). Let $\mathbf{0}$ denote the vector of all zeros and the null matrix of suitable dimension(s), and $\mathbf{1}_n \in \mathbb{R}^n$ denote the vector containing all ones. The $n \times n$ identity matrix is denoted by $\mathbb{I}_n$. Let $A \in \mathbb{R}^{n\times n}$ be a matrix. In case A is a positive definite (positive semi-definite) matrix, we write $A > \mathbf{0}$ ($A \geq \mathbf{0}$). Let $|A|$ denote the matrix A with all elements positive. The i-th element of vector x is denoted by x_i. A steady-state solution to system $\dot{x} = f(x)$, is denoted by $\overline{x}$, *i.e.*, $\mathbf{0} = f(\overline{x})$. Let $x \in \mathbb{R}^n, y \in \mathbb{R}^m$ be vectors, then we define $\mathrm{col}(x, y) := (x^\top \ y^\top)^\top \in \mathbb{R}^{n+m}$. Given a vector $x \in \mathbb{R}^n$, $[x] \in \mathbb{R}^{n\times n}$ indicates the diagonal matrix whose diagonal entries are the components of x and $\sin(x) := \mathrm{col}\big(\sin(x_1), \dots, \sin(x_n)\big)$.

II. PROBLEM FORMULATION

In this section, we introduce the nonlinear power system model together with the turbine-governor and wind turbine models. Then, two control objectives are presented: load frequency control and *optimal* generation (economic dispatch).

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TABLE I: Symbols

P_{ci}	Conventional power generation	X_{mi}	DFIG magnetizing reactance
P_{wi}	Wind power generation	X_{ri}	DFIG rotor reactance
P_{li}	Unknown constant load	X_{si}	DFIG Stator reactance
φ_i	Voltage angle	X_{ui}	Ratio between DFIG magnetizing and stator self-inductance
ω_i	Frequency deviation		
V_i	Voltage		
ι_{ds_i}	d component of DFIG stator current	R_{ri}	DFIG Rotor resistance
		R_{si}	DFIG Stator resistance
ι_{qs_i}	q component of DFIG stator current	ψ_i	Damping constant
		B	Susceptance
ι_{dr_i}	d component of DFIG rotor current	$\bar{E}_{fi}$	Exciter voltage
		τ_{ci}	Turbine time constant
ι_{qr_i}	q component of DFIG rotor current	H_i	Turbine inertia of wind turbine
V_{dri}	d component of DFIG rotor voltage	T_{mi}	Mechanical torque of wind turbine
V_{qri}	q component of DFIG rotor voltage	λ_i	Tip-speed ratio of wind turbine
V_{ti}	Terminal voltage of DFIG	r_i	Rotor radius of wind turbine
f_{ri}	Rotor angular speed of DFIG	$C_{Qi}(\lambda_i)$	Power coefficient of wind turbine
f_{bi}	Base speed of DFIG	ρ	Air density
v_i	Predicted term of wind speed	ξ_i	Speed regulation coefficient
$\tilde{v}_i$	Stochastic term of wind speed	$\mathcal{N}_i$	Neighboring areas of area i
τ_{pi}	Moment of inertia	$\mathcal{A}$	Incidence matrix of power network
τ_{vi}	Direct axis transient open-circuit constant		
		L^{com}	Laplacian matrix of communication
X_{di}	Direct synchronous reactance	u_{ci}	Control input for conventional generator
X'_{di}	Direct synchronous transient reactance	u_{wi}	Control input for wind turbine

A. Power Network Model

In this subsection, we discuss the model of the considered power network (see Table I for the description of the symbols used throughout the paper). The network topology is represented by an undirected and connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \mathcal{V}_c \cup \mathcal{V}_w = \{1, 2, ..., n\}$ is the set of the control areas and $\mathcal{E} = \{1, 2, ..., m\}$ is the set of the transmission lines. Specifically, the network comprises n_c conventional (synchronous) generators and n_w wind turbine generators. Then, $\mathcal{V}_c = \{1, 2, ..., n_c\}$ is the set of the control areas including conventional (synchronous) generators and $\mathcal{V}_w = \{n_c + 1, 2, ..., n\}$, with $n = n_c + n_w$, is the set of the control areas including wind turbine generators. Moreover, in analogy with [15], [16], we assume that the power network is lossless and each node represents an aggregated area of generators and loads. Let $\mathcal{A} \in \mathbb{R}^{n \times m}$ denote the incidence matrix corresponding to the network topology. Then, the dynamics of the overall network (known as *swing* dynamics) for all nodes (areas) $i \in \mathcal{V}$ are the following (see also [3], [6], [15], [16] for further details):

$$\begin{aligned} \dot{\theta} &= \mathcal{A}^\top \omega \\ \tau_p \dot{\omega} &= -\psi\omega + P - P_l - \mathcal{A}\Upsilon(V)\sin(\theta) \\ \tau_v \dot{V} &= -\chi_d E(\theta) V + \bar{E}_f, \end{aligned} \tag{1}$$

where $\omega, V : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, $P : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ is defined as $P := \mathrm{col}(P_c, P_w)$, with $P_c : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_c}$, $P_w : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_w}$ denoting the vector of the power generated by conventional and wind turbine generators, respectively, $\theta : \mathbb{R}_{\geq 0} \to \mathbb{R}^m$ denotes the vector of the voltage angles differences, $\chi_d \in \mathbb{R}^{n \times n}$ is a diagonal matrix whose diagonal elements are defined as $\chi_{di} := X_{di} - X'_{di}$, with $X_{di}, X'_{di} \in \mathbb{R}$, $\tau_p, \tau_v, \psi, P_l \in \mathbb{R}^{n \times n}$, and $\bar{E}_f \in \mathbb{R}^n$. Moreover, $\Upsilon : \mathbb{R}^n \to \mathbb{R}^{m \times m}$ is defined as $\Upsilon(V) := \mathrm{diag}\{\Upsilon_1, \Upsilon_2, ..., \Upsilon_m\}$, with $\Upsilon_k := V_i V_j B_{ij}$, where $k \sim \{i, j\}$ denotes the line connecting areas i and j. Furthermore, for any $i, j \in \mathcal{V}$, the components of $E : \mathbb{R}^m \to \mathbb{R}^{n \times n}$ are defined as follows:

$$\begin{aligned} E_{ii}(\theta) &= \frac{1}{\chi_{di}} - B_{ii}, && i \in \mathcal{V} \\ E_{ij}(\theta) &= -B_{ij}\cos(\theta_k) = E_{ji}(\theta), && k \sim \{i, j\} \in \mathcal{E} \\ E_{ij}(\theta) &= 0, && \text{otherwise.} \end{aligned} \tag{2}$$

B. Turbine-Governor Model for Conventional (Synchronous) Generators

In this subsection, we introduce the dynamics of the turbine-governor typically coupled with conventional (synchronous) generators. Specifically, we express the power generated by the (equivalent) synchronous generator $i \in \mathcal{V}_c$ as the output of a first-order dynamical system describing the behaviour of the turbine-governor, *i.e.*,

$$\tau_{ci} \dot{P}_{ci} = -P_{ci} - \xi_i^{-1}\omega_i + u_{ci}, \tag{3}$$

where $u_{ci} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is the control input and $\tau_{ci}, \xi_i \in \mathbb{R}_{>0}$. Now, we can write system (3) compactly for all nodes $i \in \mathcal{V}_c$ as

$$\tau_c \dot{P}_c = -P_c - \xi^{-1}\omega + u_c, \tag{4}$$

where $u_c : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_c}$ and $\tau_c, \xi \in \mathbb{R}^{n_c \times n_c}$.

Now, as it is customary in the power systems literature (see for instance [3], [6], [15]), we assign to the power generated by the synchronous generator $i \in \mathcal{V}_c$, the following strictly convex cost function:

$$J_i^c(P_{ci}) = \frac{1}{2} q_i P_{ci}^2 + z_i P_{ci} + c_i, \tag{5}$$

where $J_i^c : \mathbb{R} \to \mathbb{R}$, $q_i \in \mathbb{R}_{>0}$, $z_i \in \mathbb{R}$, and $c_i \in \mathbb{R}$ for all $i \in \mathcal{V}_c$.

C. DFIG-Based Wind Turbine Generator Model

In this subsection, we introduce the Doubly Fed Induction Generator (DFIG) dynamics of a wind turbine generator. In the DFIG-based wind turbine generator, two back-to-back converters including a rotor side converter and a grid side converter are used. The rotor side converter controls the rotor current, while the grid side converter controls the DC link voltage [17], [18]. Since wind speed affects the generated power of a wind turbine, it is then important to have a realistic model of the wind speed. In our model, we consider that the wind speed at each node $i \in \mathcal{V}$ is given by the sum of a predicted constant component v_i and a stochastic component $\tilde{v}_i$. Therefore, the Ito calculus framework is adopted to analyze the DFIG model with stochastic wind speed and to control the active power generated by the wind turbine. Before introducing the DFIG dynamics, we recall the definition of stochastic differential equation through the Ito calculus framework [19], [20].

Definition 1: **(Stochastic differential equation).** A stochastic differential equation (SDE) is defined as follows:

$$dx(t) = f(x,u)dt + g(x)d\beta(t), \tag{6}$$

where $f(x,u) \in \mathbb{R}^N$ and $g(x) \in \mathbb{R}^{N\times M}$ are locally Lipschitz, $x(t) \in \mathbb{R}^N$ is the state vector of the stochastic process, $u(t) \in \mathbb{R}^P$ is the input of the system and $\beta(t) \in \mathbb{R}^M$ is the standard Brownian motion vector.

Now, according to [17], [18], the dynamics of the DFIG-based wind turbine generator $i \in \mathcal{V}_w$ are given by

$$\begin{aligned}
\dot{\iota}_{dsi} &= \frac{f_{bi}}{K_i}\Big(-R_{si}X_{ri}\iota_{dsi} + (K_i + X_{mi}^2 f_{ri})\iota_{qsi} + R_{ri}X_{mi} \\
&\quad \iota_{dri} + X_{mi}X_{ri}f_{ri}\iota_{qri} + X_{ri}V_{ti} - X_{mi}V_{dri}\Big) \\
&:= h_{dsi}(x_i) + b_{si}V_{dri} \\
\dot{\iota}_{qsi} &= \frac{f_{bi}}{K_i}\Big(-(K_i + X_{mi}^2 f_{ri})\iota_{dsi} - R_{si}X_{ri}\iota_{qsi} \\
&\quad - X_{mi}X_{ri}f_{ri}\iota_{dri} - X_{mi}V_{qri} + R_{ri}X_{mi}\iota_{qri}\Big) \\
&:= h_{qsi}(x_i) + b_{si}V_{qri} \\
\dot{\iota}_{dri} &= \frac{f_{bi}}{K_i}\Big(R_{si}X_{mi}\iota_{dsi} - X_{si}X_{mi}f_{ri}\iota_{qsi} - R_{ri}X_{si}\iota_{dri} \\
&\quad + (K_i - X_{si}X_{ri}f_{ri})\iota_{qri} - X_{mi}V_{ti} + X_{si}V_{dri}\Big) \\
&:= h_{dri}(x_i) + b_{ri}V_{dri} \\
\dot{\iota}_{qri} &= \frac{f_{bi}}{K_i}\Big(X_{si}X_{mi}f_{ri}\iota_{dsi} + R_{si}X_{mi}\iota_{qsi} + (X_{si}X_{ri}f_{ri} \\
&\quad - K_i)\iota_{dri} - R_{ri}X_{si}\iota_{qri} + X_{si}V_{qri}\Big) \\
&:= h_{qri}(x_i) + b_{ri}V_{qri} \\
\dot{f}_{ri} &= \frac{1}{2H_i}\Big(T_{mi}(\tilde{v}_i) - X_{mi}(\iota_{dsi}\iota_{qri} - \iota_{qsi}\iota_{dri})\Big) \\
&:= h_{fri}(x_i) \\
P_{wi} &= -X_{ui}\iota_{qri}f_{ri} \\
&:= \zeta_i(x_i),
\end{aligned} \tag{7}$$

where $\iota_{dsi}, \iota_{qsi}, \iota_{dri}, \iota_{qri}, V_{dri}, V_{qri}, f_{ri}, \tilde{v}_i, P_{wi} : \mathbb{R}_{\geq 0} \to \mathbb{R}$, $x_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^6$ is the state vector of DFIG defined as $x_i := \mathrm{col}(\iota_{dsi}, \iota_{qsi}, \iota_{dri}, \iota_{qri}, f_{ri}, \tilde{v}_i)$, and $b_{si}, b_{ri} \in \mathbb{R}$ are defined as $b_{si} := -\frac{f_{bi}}{K_i}X_{mi}$ and $b_{ri} := \frac{f_{bi}}{K_i}X_{si}$. Also, $h_{dsi}, h_{qsi}, h_{dri}, h_{qri}, h_{fri}, \zeta_i : \mathbb{R}^6 \to \mathbb{R}$, $V_{ti}, f_{bi}, X_{mi}, X_{ri}, X_{si} \in \mathbb{R}$, $R_{si}, R_{ri}, H_i \in \mathbb{R}_{>0}$, and $K_i \in \mathbb{R}$ is defined as $K_i := X_{si}X_{ri} - X_{mi}^2$. Moreover, $T_{mi} : \mathbb{R} \to \mathbb{R}_{\geq 0}$ is defined as $T_{mi}(\tilde{v}_i) := \frac{1}{2}\rho\pi r_i^3 C_{Qi}(\lambda_i)(v_i + \tilde{v}_i)^2$ with $v_i \in \mathbb{R}$, $\lambda_i, \rho, r_i \in \mathbb{R}_{>0}$, $C_{Qi} : \mathbb{R}_{>0} \to \mathbb{R}_{>0}$. Now, let the stochastic term of wind speed $\tilde{v}_i$ be modeled by a SDE as in [14], *i.e.*,

$$d\tilde{v}_i = -\mu_{wi}\tilde{v}_i dt + \sigma_{wi}\tilde{v}_i d\beta, \ \forall i \in \mathcal{V}_w, \tag{8}$$

where μ_{wi} and σ_{wi} are positive constant parameters. Then, we can rewrite (7) and (8) compactly for all nodes $i \in \mathcal{V}_w$ as

$$\begin{aligned}
dx &= (H_g(x) + B_u u_w)dt + G(x)d\beta(t) \\
P_w &= \zeta(x),
\end{aligned} \tag{9}$$

where $x : \mathbb{R}_{\geq 0} \to \mathbb{R}^{6n_w}$ is defined as $x := \mathrm{col}(x_1, \ldots, x_{n_w})$, $u_w : \mathbb{R}_{\geq 0} \to \mathbb{R}^{2n_w}$ with $u_{wi} : \mathbb{R}_{\geq 0} \to \mathbb{R}^2$ defined as $u_{wi} := \mathrm{col}(V_{dri}, V_{qri})$, $\beta : \mathbb{R}_{\geq 0} \to \mathbb{R}^{6n_w}$ is the standard Brownian motion vector. Furthermore, $H_g : \mathbb{R}^{6n_w} \to \mathbb{R}^{6n_w}$ is defined as $H_g(x) := \mathrm{col}\left(H_{g1}, \ldots, H_{gn_w}\right)$ with $H_{gi}(x_i) := \mathrm{col}\big(h_{dsi}(x_i), h_{qsi}(x_i), h_{dri}(x_i), h_{qri}(x_i), h_{fri}(x_i), -\mu_i\tilde{v}_i\big)$, $G : \mathbb{R}^{6n_w} \to \mathbb{R}^{6n_w \times 6n_w}$ is defined as $G(x) := \mathrm{blockdiag}\left(G_1, \ldots, G_{n_w}\right)$ with $G_i(x) := \mathrm{diag}(0,0,0,0,0,\sigma_{wi}\tilde{v}_i)$, $\zeta : \mathbb{R}^{6n_w} \to \mathbb{R}^{n_w}$ is defined as $\zeta(x) := \mathrm{col}(\zeta_1, \ldots, \zeta_{n_w})$ and $B_u \in \mathbb{R}^{6n_w \times 2n_w}$ is defined as $B_u := \mathrm{blockdiag}\left(B_{u1}, \ldots, B_{un_w}\right)$ with $B_{ui} := \mathrm{col}\big((b_{si} \ \ 0), (0 \ \ b_{si}), (b_{ri} \ \ 0), (0 \ \ b_{ri}), \mathbf{0}_{2\times 2}\big)$.

Now, we assign to the power generated by the wind turbine $i \in \mathcal{V}_w$, the following strictly concave utility function:

$$J_i^w(P_{wi}) = -\frac{1}{2}q_i P_{wi}^2 + z_i P_{wi} + c_i, \tag{10}$$

where $J_i^w : \mathbb{R} \to \mathbb{R}$, $q_i \in \mathbb{R}_{>0}$, $z_i \in \mathbb{R}$, and $c_i \in \mathbb{R}$ for all $i \in \mathcal{V}_w$. Note that q_i and z_i are selected in order to take into account the value of the maximum power that the wind turbine can generate given the predicted wind speed v_i.

D. Control Objectives

In this subsection, we introduce the main control objectives of this work. The first objective concerns the asymptotic regulation of the frequency deviation to zero, *i.e.*,

Objective 1: **(Load Frequency Control).**

$$\lim_{t\to\infty} \omega(t) = \mathbf{0}_n. \tag{11}$$

Besides improving the stability of the power network by regulating the frequency deviation to zero, advanced control strategies additionally aim at reducing the costs associated with the power generated by the conventional synchronous generators and increasing the utilities associated with the power generated by the wind turbines. Therefore, we introduce the following optimization problem:

$$\begin{aligned}
&\min_P \ J(P) \\
&\text{s.t.} \ \sum_{i\in\mathcal{V}} \bar{P}_i - P_{li} = 0,
\end{aligned} \tag{12}$$

where $J(P) = \sum_{i\in\mathcal{V}_c} J_i^c(P_{ci}) - \sum_{i\in\mathcal{V}_w} J_i^w(P_{wi}) = \frac{1}{2}P^\top QP + Z^\top P + \mathbf{1}_n^\top C$ with $J_i^c(P_{ci})$, $J_i^w(P_{wi})$ given by (5), (10), respectively, Also, $Q \in \mathbb{R}^{n\times n}$, $Z, C \in \mathbb{R}^n$ are defined as $Q := \mathrm{diag}(q_1, \ldots, q_{n_c}, q_{n_c+1}, \ldots, q_{n_c+n_w})$, $Z := \mathrm{col}(z_1, \ldots, z_{n_c}, -z_{n_c+1}, \ldots, -z_{n_c+n_w})$, $C := \mathrm{col}(c_1, \ldots, c_{n_c}, -c_{n_c+1}, \ldots, -c_{n_c+n_w})$, respectively. In this regard, [6, Lemma 2], [15, Lemma 3] show that it is possible to achieve zero steady-state frequency deviation and simultaneously minimize the objective function $J(P)$ in (12) when the load P_l is constant. More precisely, when the load P_l is constant, the *optimal* value of P, which allows for zero steady-state frequency deviation and minimizes (at the steady-state) the objective function $J(P)$ in (12), solving the optimization problem (12), is given by:

$$P^{\mathrm{opt}} = Q^{-1}\Big(\frac{\mathbf{1}_n\mathbf{1}_n^\top(P_l + Q^{-1}Z)}{\mathbf{1}_n^\top Q^{-1}\mathbf{1}_n} - Z\Big), \tag{13}$$

where $P^{\mathrm{opt}} := \mathrm{col}(P_c^{\mathrm{opt}}, P_w^{\mathrm{opt}})$. This leads to the second objective, *i.e.*, minimization of $J(P)$ in (12), which is also known as economic dispatch or *optimal* generation [6], [15]. Then, the second goal concerning the economic dispatch or *optimal* generation is defined as follows:

Objective 2: **(Economic dispatch).**

$$\lim_{t\to\infty} P(t) = P^{\mathrm{opt}}, \tag{14}$$

with P^{opt} given by (13).

We assume now that there exists a steady-state solution to the considered power network model (1), (4) and (9).

Assumption 1: **(Steady-state solution).** There exists a constant input $(\bar{u}_c, \bar{u}_w)$ and a steady-state solution $(\bar{\theta}, \bar{\omega}, \bar{V}, \bar{P}, \bar{x})$ to (1), (4) and (9) satisfying

$$\begin{aligned}
\mathbf{0} &= \mathcal{A}^\top\bar{\omega}\\
\mathbf{0} &= -\psi\bar{\omega} + \bar{P} - P_l - \mathcal{A}\Upsilon(\bar{V})\sin(\bar{\theta})\\
\mathbf{0} &= -\chi_d E(\bar{\theta})\bar{V} + \bar{E}_f\\
\mathbf{0} &= -\bar{P}_c - \xi^{-1}\bar{\omega} + \bar{u}_c\\
\mathbf{0} &= \big(H_g(\bar{x}) + B_u\bar{u}_w\big)dt + G(\bar{x})d\beta.
\end{aligned} \tag{15}$$

Additionally, (15) holds also when $\bar{\omega} = \mathbf{0}$ and $\bar{P} = P^{\mathrm{opt}}$, with P^{opt} given by (13).

Note that the stochastic and deterministic terms of the SDE (8) at the equilibrium point are identical to zero. Thus, the steady-state solution in (12) and (15) can be considered in the deterministic sense. In the next section, we present the passivity properties for the power network, turbine-governor and wind turbine. To this end, in analogy with [6], [15], the following assumption is required:

Assumption 2: **(Steady-state voltage angle and amplitude).** The steady-state voltage $\bar{V} \in \mathbb{R}^n$ and angle difference $\bar{\theta} \in \mathbb{R}^m$ satisfy

$$\begin{aligned}
&\bar{\theta} \in (-\frac{\pi}{2}, \frac{\pi}{2})^m,\\
&\chi_d E(\bar{\theta}) - \mathrm{diag}(\bar{V})^{-1}|\mathcal{A}|\Upsilon(\bar{V})\,\mathrm{diag}(\sin(\bar{\theta}))\\
&\qquad \mathrm{diag}(\cos(\bar{\theta}))^{-1}\,\mathrm{diag}(\sin(\bar{\theta}))|\mathcal{A}|^\top\,\mathrm{diag}(\bar{V})^{-1} > 0.
\end{aligned} \tag{16}$$

Note that Assumption 2 is usually verified in practice, *i.e.*, the differences in voltage (angles) are small and the line reactances are greater than the generator reactances [6], [15].

III. Optimal Load Frequency Control

In this section, we present the passivity properties for the power network, turbine-governor and wind turbine. Then, we design a controller to achieve Objectives 1 and 2.

A. Incremental Passivity of Power Network and Turbine-Governor

In this subsection, we recall from the literature the incremental passivity of the power network and the turbine-governor model. In analogy with [15, Lemma 2], [3, Lemma 3], the incremental passivity of system (1) is obtained via the following lemma.

Lemma 1: **(Incremental passivity of system (1)).** Let Assumptions 1, 2 hold. System (1) is incrementally passive with respect to

$$\begin{aligned}
S_1 = &-\mathbf{1}_n^\top\Upsilon(V)\cos(\theta) + \mathbf{1}_n^\top\Upsilon(\bar{V})\cos(\bar{\theta}) + \frac{1}{2}V^\top DV\\
&- \big(\Upsilon(\bar{V})\sin(\bar{\theta})\big)^\top(\theta - \bar{\theta}) - \bar{E}_{fd}(V - \bar{V})\\
&- \frac{1}{2}\bar{V}^\top D\bar{V} + \frac{1}{2}(\omega - \bar{\omega})^\top\tau_p(\omega - \bar{\omega}),
\end{aligned} \tag{17}$$

and supply rate $(\omega - \bar{\omega})^\top(P - \bar{P})$, where the steady-state solution $(\bar{\theta}, \bar{V}, \bar{\omega})$ satisfies (15) and D is a diagonal matrix with $D_{ii} = \frac{1 - B_{ii}(X_{di} - X'_{di})}{X_{di} - X'_{di}}$.

Proof: The proof follows from [15, Lemma 2] and [3, Lemma 3] and is omitted due to space limitations. ■

Now, we consider the following controller proposed in [3], [6] for the turbine-governor $i \in \mathcal{V}_c$

$$\begin{aligned}
\tau_{\delta i}\dot{\delta}_i &= -\,\delta_i + P_{ci},\\
u_{ci} &= \delta_i,
\end{aligned} \tag{18}$$

where $\delta_i : \mathbb{R}_{\geq 0} \to \mathbb{R}$ and $\tau_{\delta i} \in \mathbb{R}_{>0}$. Then, in analogy with [3, Lemma 5] the incremental passivity of system (3) in closed-loop with (18) is obtained via the following lemma.

Lemma 2: **(Incremental passivity of (3), (18)).** Let Assumption 1 hold. System (3) with controller (18) is incrementally passive with respect to

$$S_{2i} = \frac{\tau_{ci}\xi_i}{2}(P_{ci} - P_{ci}^{\mathrm{opt}})^2 + \frac{\tau_{\delta i}\xi_i}{2}(\delta_i - \bar{\delta}_i)^2, \tag{19}$$

and supply rate $-(P_{ci} - P_{ci}^{\mathrm{opt}})\omega_i$, where the steady-state solution $(P_{ci}^{\mathrm{opt}}, \bar{\delta}_i)$ satisfies (15) and

$$0 = -\,\bar{\delta}_i + P_{ci}^{\mathrm{opt}}, \tag{20}$$

with P_{ci}^{opt} given by (13).

Proof: See [3, Lemma 5]. ■

B. Stochastic Passivity of DFIG-Based Wind Turbine

In this subsection, we propose a control scheme to control the active power generated by wind turbine. Then, we show that the DFIG-based wind turbine (7), (8) in closed-loop with the proposed controller is stochastically passive. Before introducing the DFIG controller, we recall the definitions of Ito derivative and stochastic passivity [19], [20].

Definition 2: **(Ito derivative).** Consider a function $S(x)$, which is twice continuously differentiable. Then, $\mathcal{L}S(x)$ denotes the Ito derivative of $S(x)$ along the SDE (6), *i.e.*,

$$\mathcal{L}S(x) = \frac{\partial S(x)}{\partial x}f(x,u) + \frac{1}{2}\mathrm{tr}\{g^\top(x)\frac{\partial^2 S(x)}{\partial x^\top\partial x}g(x)\}. \tag{21}$$

Definition 3: **(Stochastic passivity).** Consider system (6) with output $y = \eta(x)$. Assume that the deterministic and stochastic terms of the SDE (6) at the equilibrium point are identically zero, *i.e.*, $f(\bar{x}, \bar{u}) = g(\bar{x}) = \mathbf{0}$. Then, system (6) is said to be stochastically passive with respect to the supply rate $u^\top y$ if there exists a twice continuously differentiable positive semi-definite storage function $S(x)$ satisfying

$$\mathcal{L}S(x) \leq u^\top y, \ \forall (x,u) \in \mathbb{R}^N \times \mathbb{R}^P. \tag{22}$$

Now, consider the following controller for the DFIG-based wind turbine generator $i \in \mathcal{V}_w$:

$$\begin{aligned} V_{dri} =& -L_i(x_i)\big(\bar{K}_{1i}(x_i) + \bar{K}_{2i}(x_i) + \bar{K}_{3i}(x_i) + \bar{x}_i^\top \Pi_i \bar{x}_i \\ &+ \bar{x}_i^\top \Pi_i x_i + \bar{x}_i^\top \Psi_i H_{gi}(x_i)\big) \end{aligned} \tag{23a}$$

$$V_{qri} = -L_i(x_i)\big(D_{1i}(x_i)\omega_i + D_{2i}(x_i)\delta_i + D_{3i}(x_i)\big) \tag{23b}$$

$$\tau_{\delta i}\dot{\delta}_i = -\delta_i + P_{wi}, \tag{23c}$$

where

$$\begin{aligned} L_i(x_i) &= \frac{X_{ri}}{X_{ri}(\iota_{dri} - \bar{\iota}_{dri}) - X_{mi}(\iota_{dsi} - \bar{\iota}_{dsi})} \\ \bar{K}_{1i}(x_i) &= \rho\pi r_i^2 C_{Qi}((f_{ri} - \bar{f}_{ri})v_i^2 + v_i(f_{ri} - \bar{f}_{ri})^2) \\ &\quad + (f_{ri} - \bar{f}_{ri})^2 \\ \bar{K}_{2i}(x_i) &= \Big(\iota_{dsi} - \frac{X_{mi}}{X_{si}}\iota_{dri}\Big)V_{ti} + \omega_i(P_{wi} - P_{wi}^{\text{opt}}) \\ \bar{K}_{3i}(x_i) &= 2\bar{f}_{ri}X_{mi}(\iota_{dsi}\iota_{qri} - \iota_{qsi}\iota_{dri}) \\ &\quad + \Big(R_{ri}\frac{X_{mi}}{X_{ri}} + R_{si}\frac{X_{mi}}{X_{si}}\Big)\iota_{dri}\iota_{dsi} \\ &\quad + \Big(R_{ri}\frac{X_{mi}}{X_{ri}} + R_{si}\frac{X_{mi}}{X_{si}}\Big)\iota_{qri}\iota_{qsi} \\ \Pi_i &= \text{diag}\left(R_{si}, R_{si}, R_{ri}, R_{ri}, 0, 0\right) \\ \Psi_i &= \text{diag}\Big(\frac{K_i\bar{\iota}_{dsi}}{f_{bi}X_{ri}}, \frac{K_i\bar{\iota}_{qsi}}{f_{bi}X_{ri}}, \frac{K_i\bar{\iota}_{dri}}{f_{bi}X_{si}}, \frac{K_i\bar{\iota}_{qri}}{f_{bi}X_{si}}, 0, 0\Big) \\ D_{1i}(x) &= -X_{ui}\iota_{qri}f_{ri} + X_{ui}\bar{\iota}_{qri}\bar{f}_{ri}, \\ D_{2i}(x) &= (P_{wi} - P_{wi}^{\text{opt}})\delta_i \\ D_{3i}(x) &= (\delta_i - P_{wi})^2 - (P_{wi} - P_{wi}^{\text{opt}})P_{wi}^{\text{opt}}, \end{aligned}$$

Note that the controller (23) requires the information of $\bar{x}_i$ and P_{wi}^{opt} which can be obtained by solving (15) and (13), respectively, where the loads are assumed to be constant. Also, the idea behind the design of the controller (23) is using passivity based control method. In order to obtain the stochastic passivity of (7), (8), (23), we need to consider the following assumptions on the wind turbine and speed.

Assumption 3: **(Condition on the rotational speed).** The rotational speed f_{ri} of the wind tubine $i \in \mathcal{V}_w$ is bounded as $|f_{ri}| < \bar{\gamma}_i$, $\bar{\gamma}_i \in \mathbb{R}_{>0}$.

Assumption 4: **(Condition on the parameters of** (8)**).** The wind speed parameters in (8) satisfies

$$\mu_{wi} + \bar{f}_{ri} > \frac{\sigma_{w_i}^2}{2} + v_i + \bar{\gamma}_i, \quad i \in \mathcal{V}_w. \tag{24}$$

Note that Assumption 3 is true in practice, since the rotational speed of a wind turbine is limited by the mechanical characteristics of the turbine itself, which is indeed usually equipped with mechanical breaks that avoid high rotational speed. Assumption 4 is instead a sufficient technical condition to establish the stochastic passivity of the wind turbine.

Now, the stochastic passivity of DFIG-based wind turbine dynamics (7), with wind speed dynamics (8) and controller (23) is obtained via the following proposition.

Proposition 1: **(Stochastic passivity of** (7)**,** (8)**,** (23)**).** Let Assumptions 3 and 4 hold. System (7), (8) in closed-loop with (23) is stochastically passive with respect to

$$\begin{aligned} S_{3i} =& \frac{K_i}{2f_{bi}X_{ri}}\Big((\iota_{dsi} - \bar{\iota}_{dsi})^2 + (\iota_{qsi} - \bar{\iota}_{qsi})^2 \\ &+ (\iota_{dri} - \bar{\iota}_{dri})^2 + (\iota_{qri} - \bar{\iota}_{qri})^2\Big) + 2H_i(f_{ri} - \bar{f}_{ri})^2 \\ &+ \rho\pi r_i^3 C_{Qi}\tilde{v}_i^2 + \frac{\tau_{\delta i}}{2}(\delta_i - \bar{\delta}_i)^2, \end{aligned} \tag{25}$$

and supply rate $-\omega_i(P_{wi} - P_{wi}^{\text{opt}})$, where the steady-state solution $(\bar{x}_i, P_{wi}^{\text{opt}}, \bar{\delta}_i)$ satisfies (15) and

$$0 = -\bar{\delta}_i + P_{wi}^{\text{opt}}, \tag{26}$$

with P_{wi}^{opt} given by (13).

Proof: See [21, Proposition 1]. ∎

C. Closed-loop analysis

In this subsection, we show that the closed-loop system is stochastically stable. First, we recall the definition of (asymptotic) stochastic stability [19], [20].

Definition 4: **((Asymptotic) stochastic stability).** System (6) is (asymptotically) stochastically stable if a twice continuously differentiable positive definite Lyapunov function $S : \mathbb{R}^N \longrightarrow \mathbb{R}_{>0}$ exists such that $\mathcal{L}S$ is (negative definite) negative semi-definite.

Now, in order to achieve Objective 2, we modify controllers (18) and (23c) as follows (see [3], [6]):

$$\tau_\delta\dot{\delta} = -\delta + P - \text{blockdiag}(\xi^{-1}, \mathbb{I}_{n_w})QL^{\text{com}}(Q\delta + Z), \tag{27}$$

where $\delta : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, $\tau_\delta \in \mathbb{R}^{n\times n}$ and $L^{\text{com}} \in \mathbb{R}^{n\times n}$ is the Laplacian matrix associated with a connected communication network. More precisely, the term $Q\delta + R$ in (27) reflects the marginal cost associated with $J(P)$ in (12) and $L^{\text{com}}(Q\delta + Z)$ represents the exchange of such information among different areas. In the following theorem, we show that the closed-loop system (1), (4), (9), (23a), (23b), (27) is stochastically stable and Objectives 1 and 2 are attained.

Theorem 1: **(Closed-loop analysis).** Let Assumptions 1–4 hold. Consider system (1), (4), (9) with controller (23a), (23b), (27). Then, the solutions to the closed-loop system starting sufficiently close to $(\bar{\theta}, \bar{\omega} = \mathbf{0}, \bar{V}, P^{\text{opt}}, \bar{x}, \bar{\delta})$ stochastically converge to the set where $\bar{\omega} = \mathbf{0}$ and $\bar{P} = P^{\text{opt}}$, *i.e.*, achieving Objectives 1 and 2.

Proof: See [21, Theorem 1], where the proof follows from Lemmas 1, 2, Proposition 1 and LaSalle's principle. ∎

IV. Simulation Results

In this section, the simulation results show excellent performance of the proposed control scheme. We consider a power network partitioned into four control areas which is shown in [22, Fig. 1], where areas 1, 2 and 3 include conventional generation, while area 4 includes wind generation. The system parameters are provided in [21, Table II], where the parameters are equal to [17, Table I] and [22, Table II],

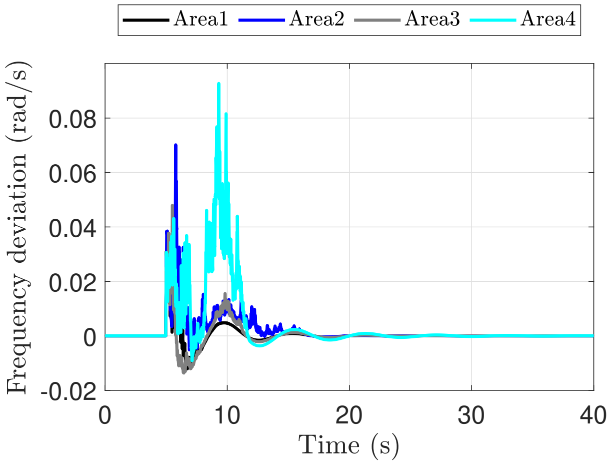

Fig. 1: Frequency deviation in each area.

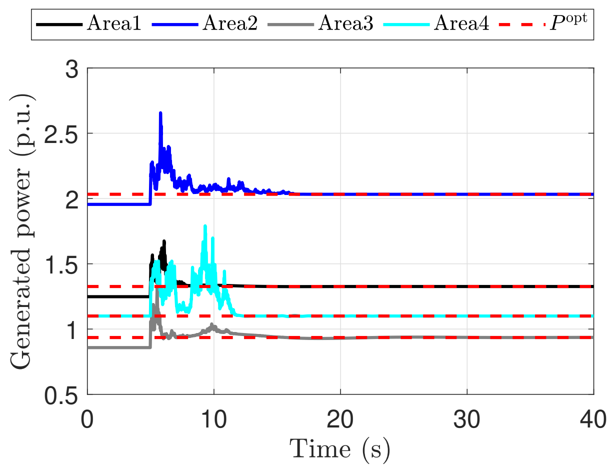

Fig. 2: Generated power in each area.

the nominal frequency and power base are chosen equal to 120π rad/s and 1000 MVA, respectively.

The system is initially at the steady-state with constant load $P_l = \text{col}(1.3, 2, 1.3, 0.5)$. Then, at the time instant $t = 5$ s the load increases to $P_l = \text{col}(1.4, 2.1, 1.4, 0.55)$ and the wind speed varies according the stochastic differential equation (8). Fig. 1 shows that the frequency deviation in each area converges to zero after a transient time. Also, we notice from Fig. 2 that after $t = 5$ s the generated power in each area converges to the corresponding optimal value. Specifically, we observe that the additional power demand is supplied by the conventional generators while the wind turbine (Area 4) generates the maximum possible power given a certain wind speed.

V. Conclusion

In this paper, we have verified the (stochastic) passivity of the power network including conventional synchronous generators and wind turbines, where the wind speed is described by a stochastic differential equation and presented a distributed control scheme to ensure the stochastic stability of the system, achieving optimal load frequency control.

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