A Survey on Event-Triggered Sliding Mode Control

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Abstract—Event-triggered controllers are well known for guaranteeing the desired stability for a sampled-data system with minimum resource utilization. Over the past decade, the study has revealed that the overall performance improvement for a sampled-data system can be achieved by replacing the time-based sampling with an event-triggered one. The design of sliding mode control (SMC) in the event-triggering framework has also shown similar outcomes, especially for uncertain systems. There are different design strategies for event-triggered SMC available in the literature for networked dynamical systems that are potentially affected by uncertainties and transmission delays. The purpose of this survey paper is to present the state of the art on event-triggered SMC and familiarize the readers with the design techniques with their pros and cons, since this will be very helpful to the researchers and engineers for implementing SMC using event based feedback strategies.

Index Terms—Event-triggered control, practical sliding mode, sampled-data system, sliding mode control.

I. INTRODUCTION AND MOTIVATION

VARIABLE structure systems via sliding mode originated in the then Soviet Union in the late fifties as a new robust control technique for uncertain systems [1]. Stabilization of plant is achieved by the action of a discontinuous control law which forces the trajectories to slide along a predesigned (stable) manifold. Since then, this technique has been regarded as an efficient method to achieve robust stability for a wide class of systems including many practical applications. The highly cited survey articles and excellent books that appeared in the literature are testimony to this fact; the important ones can be found in [2]–[9] which still serve as a reference guide for beginners. The research on sliding mode control (SMC) is actively pursued on the broad areas that focus on different aspects of this control technique. It can be loosely said that the classical SMC [1], [6], chattering alleviation [10], [11], high-order SMC [7], [8], [12], terminal SMC [13], [14], discrete SMC [15], [16] are the broad topics of investigation although they are highly interrelated.

The purpose of the current paper is to introduce the readers to a new area related to the digital implementation of SMC, the event-triggered SMC, which is getting more popular among researchers as it is witnessed by more than 130 articles found in the literature in just a short period after its first publication [17], [18] (as of 15th October 2020). The motivation of studying the event-triggered SMC stems from the discrete SMC. In the eighties, the study of discrete SMC was started to account for the integration of powerful sliding mode technique with the digital processors. Since then the discrete SMC has seen considerable effort among researchers (see [19]–[21] for important contributions). The interesting feature is that no exact sliding motion takes place due to the unavailable of switching action along the sliding manifold, and this has led to a new motion known as the quasi sliding mode [19]. Till date there has been a constant interest in how to reduce the band size of the quasi sliding motion. Although there are many approaches to this problem, the most important one is the implicit discretization proposed in [22], where the numerical chattering can be reduced significantly to an acceptable level. All the techniques presented in the literature consider the application of control law periodically. In areas such as network-based control, this periodic transmission/execution may not be very appreciable owing to its serious limitations. A new discrete implementation strategy known as the event-triggered control (ETC) is proposed to overcome the drawbacks of existing discrete design [23], [24].

One of the very important works on ETC is the Lebesgue sampling considered in [23] for a first-order stochastic system where some superior performance compared to the Riemann (periodic) sampling is guaranteed. Later, the author of [24] presented the design of ETC for a nonlinear system. The study of event based design of SMC was first reported in [17], [18]. In [18], an integral sliding mode (ISM) based model predictive controller was designed whereas the design with classical SMC was presented in [17]. Thereafter, many results have been reported on the event-triggered SMC which constitutes the new findings in this area. The implementation of SMC through an event based sampling achieves the practical sliding mode, i.e., any desired band size around the sliding manifold can be obtained. This new notion of practical sliding mode was introduced in [17] (and extended to second-order SMC in [25], [26], which is however not found in the periodic discrete design of SMC where reducing the band below a certain value is a challenging task. This is one of the remarkable outcomes of the event-triggered SMC. Yet, there are many avenues for future research in this field, particularly related to high-order sliding mode and robust differentiators (see Section VI for details).

It is our observation that there has been considerable interest in this topic among researchers. The works particularly range from the significant theoretical contribution to the study of useful practical problems. To name a few are the network-based...
control [27], multi agent systems [28], power converters [29], etc. where the event based SMC was successfully designed for specific applications. This survey will provide to the readers the state of art on this topic starting from its basic design philosophy to the detailed applications. The presentation has been kept simple so that it can be grasped without much background on it.

The organization of the paper is made as follows: Section II briefly presents the idea of ETC. In Section III, we review the results of the event based design of SMC for linear systems. The results of the nonlinear system are discussed in Section IV. Different applications of the event-triggered SMC are presented in Section V. Finally, Section VI collects the concluding remarks of this paper along with some future scopes to continue further research in this area.

II. WHAT IS AN EVENT-TRIGGERED CONTROLLER?

Designing a controller for a sampled-data system has been an important area of research since its first introduction. Here, the feedback is restricted only to the discrete time instants, and hence the performance of the closed-loop system may be greatly affected. One of the common approaches used in dealing with this problem is to consider an ideal sampler and a zero-order hold (ZOH), which both work in a synchronized manner under a clock-based sampling strategy. The sampled-data controller under this sampling is known as time-triggered control, and this has been a focus of many researchers for a long time. Replacing the time-based sampling with a system dependent sampling (the sampling mechanism can depend on the state and/or other system parameters), some additional benefits can be achieved in the overall performance. Here, the system parameters and signals determine the output of the sampling function, and we call this type of sampling-based controller an ETC [23].

For the exposition of the idea of ETC, we consider a linear-time invariant (LTI) system as

\[
\dot{x} = Ax + Bu + d, \quad x(0) = x_0,
\]

where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R} \) and \( d \in \mathbb{R} \) are the control and disturbance inputs, respectively; the matrices \( A \) and \( B \) are of appropriate dimensions. Let the following assumptions hold for the system:

**Assumption 1:** The matrix pair \( (A, B) \) is controllable.

**Assumption 2:** There exists a known constant \( d_0 > 0 \) for which it holds that \( \sup_{t \geq 0} |d(t)| \leq d_0 \).

These two assumptions are very standard in the sliding mode literature. As our plant is to be controlled in the sampled-data setting, we consider an ideal sampler located in the sensor end which sends the sampled information, and ZOH in the actuator side to implement the control input as shown in Fig. 1. The control problem is to stabilize the origin of the closed-loop system in the presence of disturbance under this setup.

Let \( \{t_i\}_{i \in \mathbb{Z}_{\geq 0}} \) be a sequence of sampling/triggering instances generated by an event-triggering mechanism. Specifically, an event condition is employed at the sensor side to generate the sampling sequence. Although the implementation of ETC is complex than the time-triggered one, it can reduce the frequency of transmission of information over the network and improve the overall performance. Indeed, the time sequences can be generated by:

\[
t_{i+1} = \inf \{t > t_i : T(e(t)) \geq 0\}, \quad t_0 = 0,
\]

where \( T(e(t)) \) is the event function that depends on the sampling error \( e(t) = x(t_i) - x(t), \quad t \in [t_i, t_{i+1}) \). The key aspect of the event-triggering mechanism is that the inter-event time \( T_i := t_{i+1} - t_i \) must satisfy \( T_i \geq \tau_T \) for all \( i \in \mathbb{Z}_{\geq 0} \) and some constant \( \tau_T > 0 \). The triggering mechanisms satisfying the above property are free from an undesired behavior called the Zeno phenomenon [30]. All the ETC proposed in the literature guarantee the existence of no Zeno behavior in the control execution.

In an ETC, the goal is to design a sampled-data controller which is applied to the plant through an event-triggering strategy. Generally, the event conditions depend on the system parameters and its trajectory. Keeping this in mind, one may write the expression for control law as

\[
u(t) = u(t_i), \quad \forall t \in [t_i, t_{i+1}), \quad i \in \mathbb{Z}_{\geq 0}.
\] (2)

This controller is known as the ETC, which is simply a piecewise constant function between two event instants. The closed-loop system under the ETC guarantees the desired stability and admits a uniform lower bound for the sampling sequence. The event functions mostly depend on the nature of the controller and the stability tools used to show the desired property. In the context of SMC, a new sliding motion is established by the controller which is completely characterized by the triggering condition parameters. We postpone these discussion to the next section as it requires some more technical details.
A few well known existing ETC strategies are discussed here. The initial works in [23], [31] presented some performance improvement by implementing an event-based sampling. The author in [24] considered the design of the sampling mechanism in the input-to-state stability framework. Since then a large number of works have appeared in the literature and we refer the readers to [32]–[35] and the references therein for significant developments on the topic. Moreover, an ETC approach for nonlinear uncertain systems is developed in [36] by using the notion of input-to-state stability (ISS) and the nonlinear small-gain theorem, a framework for the event-triggered stabilization of nonlinear systems using hybrid systems tools is proposed in [37], model-based periodic event-triggering mechanisms are proposed in [38], an event-triggered output feedback control law based on an high-gain observer is constructed in [39], an event-triggered $H_{\infty}$ control approach is designed in [40]. Although all these works are of utmost importance, we restrict the focus of our paper only to the event-triggered SMC approach due to space limitation.

III. EVENT-TRIGGERED SMC FOR LINEAR SYSTEMS

This section presents different event-triggered strategies of SMC for LTI systems. Here, we first introduce briefly the design of SMC and then, we categorically study the advantages and disadvantages of all these algorithms. The same dynamical system given by (1) is considered here.

Following Assumption 1, one can write the input matrix in (1) as $B = \begin{bmatrix} B_1^T & B_2^T \end{bmatrix}^T$ for any nonzero $B_2 \in \mathbb{R}$ (For an $m$-inputs system, the sub-block matrix $B_2 \in \mathbb{R}^{m \times m}$ is invertible). The coordinate transformation, $x \mapsto z := Wx$, where

$$W = \begin{bmatrix} I & -B_1B_2^{-1} \end{bmatrix},$$

transforms the system (1) into

$$\dot{z} = \bar{A}z + \bar{B}(u + d), \quad (3)$$

where the matrices $\bar{A}$ and $\bar{B}$ are of the form

$$\bar{A} = WAW^{-1} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}$$

and $\bar{B} = WB = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, with their partition of the state vector $z = \begin{bmatrix} z_1^T \\ z_2^T \end{bmatrix}^T \in \mathbb{R}^n$, and $z_1 \in \mathbb{R}^{n-1}$ and $z_2 \in \mathbb{R}$. The above representation in (3) is called the regular form in the sliding mode literature (see [5]). In this coordinate, only a few components of the system dynamics are affected by the unknown disturbance while the rest of it is described by the known components.

Note that the matrix pair $(\bar{A}_{11}, \bar{A}_{12})$ is controllable provided Assumption 1 holds. Using this fact, the sliding hyperplane can be designed such that the trajectory of the system dynamics on this plane converges to the origin. Let $C_1^T \in \mathbb{R}^{n-1}$ be any gain such that $\bar{A}_{11} - \bar{A}_{12}C_1$ is Hurwitz. Then, construct the sliding variable $s = Cz$ for $C^T = \begin{bmatrix} C_1 & 1 \end{bmatrix}^T \in \mathbb{R}^n$, and corresponding to this, a manifold is defined as

$$S := \{z \in \mathbb{R}^n : s = Cz = 0\}. \quad (4)$$

It can be noted that the trajectories restricted to the set $S$ renders a stable motion, i.e., the plant trajectories converge to the origin irrespective of the disturbance. The goal of a sliding mode based controller is to steer the closed-loop system trajectories to the sliding manifold in a finite time and eventually force them to slide along it. This motion is known as the sliding mode. Since ETC is applied in a discrete fashion through ZOH, it is not possible to obtain a sliding motion same as in the analog-based implementation. In this case, the trajectories are confined to a predefined boundary around the sliding manifold $S$, and in steady-state, it becomes bounded around the equilibrium point. This motion is often termed in the context of event-triggering based SMC as the practical sliding mode, and this is recalled below for completeness of our presentation.

Definition 1 ([41]): A system is said to be in practical sliding mode if for any $\varepsilon > 0$, there exists a $\tau > 0$ such that $|x(t)| \leq \varepsilon$ for all $t \geq \tau$.

Different event-triggered algorithms in SMC were proposed for LTI systems in the literature that show practical sliding motion in the closed-loop system [17], [34], [42]–[44]. There are many criteria based on which the event-triggered algorithms can be categorized into different types. One such factor is the nature of stability it guarantees for the closed-loop system.

A. Event Conditions Based on Stability Criteria

The event-triggered algorithms for SMC can be classified based on the different stability criteria. An event-triggered mechanism may be called the global if the equilibrium point can be stabilized globally by the respective sampling strategy and the inter event times are uniformly lower bounded from zero globally. Similarly, it can be categorized into the semi-global and local event-triggered algorithms. The readers are referred to [41], [43] for the different notions of event-triggering strategy.

1) Global Event-Triggered SMC: The authors of [43], [45] proposed an ETC law for achieving the global stability. Here, it is shown that the control law

$$u(t) = -CAz(t_i) - K(z(t_i))sign(z(t_i)), \quad (5)$$

where $K(z(t_i)) = K_1||z(t_i)|| + K_2$ with $K_1 > 1$ and $K_2 > d_0$, implemented with the event-triggering mechanism

$$t_{i+1} = \{t > t_i : \|C||\bar{A}||e(t)|| \geq \sigma(\alpha_1||z(t_i)|| + \alpha_2)\} \quad (6)$$

for any $\sigma \in (0, 1)$, positive scalars $\alpha_1$ and $\alpha_2$ and $e(t) = z(t_i) - z(t)$, can ensure that the closed-loop plant trajectories converge to the vicinity of $z = 0$ globally. As the switching gain is a function of the sampled value of the state, the steady-state bound also depends on the sampled value of state trajectory. This is the price paid for achieving stability globally. However, one may still recover the practical sliding motion, i.e., any steady-state bound of closed-loop plant trajectories can be achieved, by switching the global event-triggered strategy to a semi-global (or local) one appropriately.

2) Semi-Global Event-Triggered SMC: In the LTI system, the local ETCs are not studied much because of their less relevant. In fact, it can be thought of as a special case of the semi-global event-triggered strategies. To design a semi-global event-triggered SMC, we consider a bounded region
of arbitrary size. One may follow the steps given below to construct the set [46].

Let $D_1 = \{ z_1 \in \mathbb{R}^{n-1} : z_1^T P_1 z_1 \leq c_1 \}$ and $D_2 = \{ s \in \mathbb{R} : |s| \leq c_2 \}$ be two sets for some positive constants $c_1$ and $c_2$, and the positive definite matrices $P_1$ and $Q_1$ satisfying the equation $(\overline{A}_{11} - \overline{A}_{12} C_1^T) + P_1 + P_1 (\overline{A}_{11} - \overline{A}_{12} C_1^T) = -Q_1$. Choose the constants $c_1$ and $c_2$ satisfying

$$2\sqrt{\frac{\| P_1 \overline{A}_{12} \|}{\lambda_{\min} (Q_1)}} \leq \sqrt{\frac{c_1}{\lambda_{\max} (P_1)}},$$

and then construct the set

$$D = \left\{ W^{-1} \begin{bmatrix} z_1 & 1 \end{bmatrix} \in \mathbb{R}^n : (z_1, s) \in D_1 \times D_2 \right\},$$

where $W = \begin{bmatrix} I_{n-1} & 0 \\ C_1 & 1 \end{bmatrix}$. By taking the scalars $c_1$ and $c_2$ appropriately (as above), the set $D$ can be designed to include any arbitrary bounded region. Thus, one can show the semi-global stability over this domain. The main purpose of such construction of the set is to ensure that it remains positively invariant, i.e., the trajectories do not go out of this region for any arbitrary bounded region. Thus, one can show the semi-global stability over this domain. The main purpose of such construction of the set is to ensure that it remains positively invariant, i.e., the trajectories do not go out of this region for any (future) time once it starts there. In [17], [41], [46], an extensive analysis was presented for the design of semi-global event-triggering based SMC.

The event-triggered sampling strategy was proposed in [41] which is given as

$$t_{i+1} = \{ t > t_i : ||C|| ||\overline{A}|| ||e(t)|| \geq \sigma \alpha \}, \quad (7)$$

where $\sigma \in (0, 1)$ is a parameter to tighten the event condition further, and $\alpha > 0$ is the design parameter. The corresponding control law is designed as

$$u(t) = -C \overline{A} z(t_i) - K \text{sign}(s(t_i)), \quad \forall t \in [t_i, t_{i+1}) \quad (8)$$

for any $K > \alpha + d_0$. Unlike the global case, here the switching gain and the event condition depend on the constant threshold ($\alpha$) which can be designed priori to meet the performance requirements. Also, as a result of this, the semi-global triggering algorithm achieves practical sliding mode in the system. One of the interesting features of the semi-global algorithm was reported in [47]. Here, it was shown that the stability is preserved for any given $\alpha > 0$ in the closed-loop system although high values of $K$ and $\alpha$ may result in a larger size of the practical sliding mode.

**B. Event-Triggered SMC Based on Sampled Measurements**

There is yet another way of designing an event-triggering based design of SMC using only discrete measurements of plant states. The major advantage of this method is to combine the existing well known time-triggered sampling mechanism with the event-triggered one. One of the popular techniques, in this case, is the discrete event-triggering where the event conditions are verified at discrete instants. The control signal can be applied to the plant on the occurrence of the event. In this case, a discrete model of the plant is used to design such an event based controller. The design of discrete event-triggered SMC was reported in [46], [48]–[53]. The event condition is similar to that of (7) but in the discrete setting. In [48], output feedback based event-triggered SMC was proposed for a linear discrete-time system using fast output sampling. A similar analysis was reported for delta operator systems in [50], and an extension of this algorithm with fast output sampling can be found in [44].

Another approach to the design of such a controller is called the periodic ETC [38], [54], [55]. Here, the event condition is verified periodically over time to generate a possible triggering time instant. Therefore, no continuous monitoring of triggering conditions is needed here, and hence the computational burden may be reduced to some extent. The periodic event-triggered SMC was reported in [56]. The triggering instants are always integer multiples of the sampling period at which the measurements of trajectory are taken. However, the most intricate part of periodic ETC is that the event condition may be satisfied before it can be verified. As a consequence of this, the event based sampling mechanism maintains a different condition than that is used in the triggering mechanism. This is the price paid to account for the periodic evaluation of the event condition.

Authors of [56] proposed an algorithm to design periodic event-triggered SMC that overcomes the technical challenges. For any given $\delta > 0$, choose the switching gain of SMC as

$$K > (1 + \delta) \alpha + d_0,$$

where $\alpha > 0$ is the triggering parameter. Then, the sampling period, $h$, for which periodic ETC can guarantee the stability is given as $h \in (0, \Xi_0)$ where

$$\Xi_0 := \inf_{z \in D} \Xi(z) = \inf_{z \in D} \frac{1}{\| A \|} \ln \left( 1 + \frac{\sigma \alpha}{\| C \| (\rho(\| z \|) + \beta)} \right)$$

for $\rho(\| z \|) = \| A - BCA \| \| z \|$ and $\beta = \| B \| (K + d_0)$. With these set of parameters, the periodic event-triggered strategy is given by

$$t_{i+1} = \inf \{ t_i + jh : ||C|| ||\overline{A}|| ||e(t_i + jh)|| \geq \sigma \alpha, j \in \mathbb{Z}_{\geq 0} \}.$$

One may observe that the periodic event-triggered SMC ensures the stability on some given domains of interest. This is because the lower bound of the sampling period can be obtained only on a given domain. Yet, the novelty of the above algorithm remains in the fact that the practical sliding mode can still be achieved with only periodic evaluation.

The periodic event-triggering transmission was considered in [27] for the control of a network based uncertain stochastic system. For a sampling period $h > 0$, the triggering sequence is generated by the satisfaction of the condition

$$\mathbb{E}[e_y^T (t_i + jh) \Delta_1 e_y(t_i + jh)] > \rho \mathbb{E}[y^T (t_i + jh) \Delta_1 y_t(t_i + jh)]$$

where $e_y(t) = y(t_i) - y(t)$ is the output sampling error, $\mathbb{E}[\cdot]$ is the expectation operator and $\Delta_1 > 0$ is a positive definite matrix. At the time of event generation, the output data are transmitted to the controller that achieves an $H_\infty$ performance level for the closed-loop plant.
C. Variants of Event-Triggered SMC

There are a few variants of event-triggered strategies found in the literature [41], [57]–[59]. These methods implement ETCs by modifying the triggering strategies in such a way that certain drawbacks/conservatisms can be avoided/minimized. We emphasize here the self-triggering [57], [60], decentralized event-triggering [58], dynamic event-triggering [59], [61] to highlight the respective advantages. In each of these techniques, some additional benefits are reported which is otherwise not possible in the classical event-triggered algorithm.

In self-triggering based implementation, the triggering times are generated from the lower bound estimate of the inter event time. Thus, the sampling instants can be generated by

\[ t_{i+1} = t_i + \Xi(z(t_i)). \]

As there is no continuous evaluation needed in this scenario, many advantages in practical implementation can be obtained, such as power saving in the onboard implementation, lesser computation, etc. However, it does not yield any improvement in terms of larger inter-event time. In the decentralized ETC, a distributed event condition is designed that can be evaluated at each sensor node in a decentralized manner. The control signal can be updated when a triggering time is generated by any of the distributed event conditions [58].

The dynamic event-triggered SMC is another variant that is used to obtain a sparser triggering sequence. In [59], a dynamic ETC with an observer-based integral sliding mode is presented. The control is applied in an analog fashion in their analysis, unlike the ZOH based implementation. Moreover, the sampled measurements are taken into consideration in their design. There is another way of designing dynamic event-triggered SMC developed in [62] under ZOH based implementation of the control law. Moreover, the proposed strategy guarantees a larger inter-event time than that of the (static) ETC ensuring a sparser sampling sequence.

Apart from the above techniques, there are also a few other variants that stand with their own merit. For example, implementation of ETC via reduced-order controller design reported in [46]. Due to the reduced order triggering mechanism, the sampling sequence becomes more sparse when one compares it in an appropriate coordinate. The most important outcome of this strategy is that a reduced order vector can be transmitted over the network; thus, transmission cost becomes less. Another variant was proposed in [63] based on the reaching law approach. Here, a time-varying event-triggered condition was proposed to show the faster convergence to the sliding hyperplane. The event-triggered based integral sliding mode was also developed with the in [18], [64]–[66].

The above event-triggered strategies do not consider measurement noise, which is a notorious open problem in SMC theory. Specifically, in the presence of noise, the trajectories cannot be bounded within any arbitrary bound. However, by assuming a certain bound for noise, one can ensure the ultimate boundedness of the trajectories. In this survey paper, we focus only on the noise-free case because it is the most studied and analyzed in the literature, where the control law guarantees the practical sliding motion, which is the main feature of the event-triggered SMC.

IV. EVENT-TRIGGERED SMC FOR NONLINEAR SYSTEMS

This section describes the state of art in the event-triggering based design of SMC for nonlinear systems. We mainly focus here on specific nonlinear models that were already considered in the literature. There are broadly two ways of designing ETC for nonlinear cases. Although both the methods ensure similar results, the philosophy of designing the controller is different.

Consider the following class of nonlinear model [34], [67]:

\[
\begin{align*}
\dot{x}_1 &= f_1(x_1) + B_1 x_2 \\
\dot{x}_2 &= f_2(x_2) + B_2 u + B_2 d
\end{align*}
\]

(9a) where \( x = \begin{bmatrix} x_1^T & x_2^T \end{bmatrix}^T \in \mathcal{X} \subset \mathbb{R}^n \) is the state vector with \( x_1 \in \mathbb{R}^{n-1} \) and \( x_2 \in \mathcal{U} \), \( u \in \mathcal{U} \subset \mathbb{R} \) is the control input. The sets \( \mathcal{X} \) and \( \mathcal{U} \) are open and they contain the origin in their respective coordinate. Like in the linear case, it can be assumed that disturbance is bounded with a priori known bound, \( d_0 \).

In addition, the assumptions imposed on the above system are that the vector fields are Lipschitz over compact domains, and \( f_1(x_1) = A_1 x_1 + \gamma(x_1) \) with \( \gamma(\cdot) \) being a continuous function satisfying \( ||\gamma(x_1)||/||x_1|| \to 0 \) as \( x_1 \to 0 \). Choose any \( C_1^i \in \mathbb{R}^{n-1} \) such that \( A_1 - B_1 C_1 \) is Hurwitz. Based on this, one may design the sliding variable as \( s = C x \) where \( C = \begin{bmatrix} C_1 & 1 \end{bmatrix}^T \in \mathbb{R}^n \), and the associated sliding manifold as

\[
\mathcal{S} = \{ x \in \mathcal{X} : s = C x = 0 \}.
\]

Like in the linear case, here also ETC does not render the sliding motion in the sense of analog implementation (see [34] for a detailed discussion). However, one can still achieve a practical sliding mode by appropriately designing the control law. It is shown that the controller of the form

\[
\begin{align*}
u(t) &= -B_2^{-1}(C f(x(t)) + K \text{sign}(s(t)))
\end{align*}
\]

(10) for \( K > \alpha > |B_2|d_0 \), along with the event-triggered strategy

\[
t_{i+1} = \{ t > t_i : L ||C|| ||e_x(t)|| \geq \sigma \alpha \},
\]

where \( \alpha > 0 \) is any given constant, establishes the practical sliding mode in the closed-loop system. Here, \( L > 0 \) is the Lipschitz constant for the vector field \( f(x) = \begin{bmatrix} f_1(x_1) & B_1 x_2 & f_2(x) \end{bmatrix}^T \) and \( e_x(t) = x(t_i) - x(t) \) is the sampling error in the system. Note that the ETC is developed on a given domain only.

The other method of designing the ETC for the nonlinear systems is based on the use of sliding variable in the event condition [25], [26], [68]–[70]. The event is generated only when the magnitude of the sliding variable equals a given threshold, and the plant runs with this new control input until the next triggering instant. One of the advantages of this kind of event based sampling is that the inter-event time may be larger. Consider the following system [68]:

\[
\begin{align*}
\dot{x}_i &= x_{i+1}, \quad i = 1, 2, \ldots, n - 1, \\
\dot{x}_n &= f(x, t) + b(x, t)u + d
\end{align*}
\]

(11) where \( x = [x_1 \ x_2 \ \cdots \ x_n]^T \in \mathcal{X} \) is the state vector, \( f : \mathcal{X} \times \mathbb{R} \to \mathbb{R} \) and \( b : \mathcal{X} \times \mathbb{R} \to \mathbb{R} \) are bounded uncertain functions, and \( d : \mathbb{R} \to \mathbb{R} \) is the bounded external
the assumption that on the sliding manifold the origin of the system is an asymptotically stable equilibrium point and that the system admits a global normal form in the state space, i.e., there exists a global diffeomorphism that allows to reduce the analysis of the original system to the one of a simpler nonlinear system: the so-called auxiliary system [8].

In the next section, few other techniques for designing event-triggered SMC for nonlinear systems are discussed in relation to their specific applications [72]–[74].

V. APPLICATIONS

This section discusses the application of event-triggered SMC to important areas such as the networked control systems, multi-agent systems, power systems, etc.

A. Networked Control Systems

Networked control system is a very important area of application of the ETC. As the prime motivation of this technique aims at reducing the burden on the system, network-based systems can utilize event-based communication to improve its overall performance. In [18], the authors developed an ISM-based ETC via model predictive control for networked dynamical systems whereas in [75] an ETC was developed with model predictive control in using ISM. The extension to a model-based SMC over the network was reported in [76], [77]. In all the cases, a sliding variable based event condition is used to implement the controller. For stochastic systems, the network-based SMC was considered in [27], [78], [79]. As the communication channels suffer from different network constraints, such as packet loss, transmission delay, quantization, etc, the design of the event-triggering based controller is also studied for these specific cases separately. In [80], output feedback based SMC was designed for a discrete system under packet drops in the network. Recently, a lot of attention is paid to the design of quantized feedback based SMC. In an event-triggered setting, quantized feedback based SMC is studied in [42], [81], [82], for the switched system under dynamic quantization policy was analyzed in [83], for LTI systems under dynamic quantization in [84], for the mechanical system in [85]. Similarly, under the constraint of transmission delay, this problem is again studied in [34], [86] for a class of nonlinear systems, [87] for fuzzy systems, [79] for stochastic systems, [88] for discrete-time system via output feedback. Similarly, in a distributed system, decentralized ETC is formulated in [58], [89]. The other network-based event based SMC can be seen in [90]–[93].

B. Multi-Agent Systems

Another important application area of event based SMC is the multi-agent systems. In this systems, achieving a robust performance is one of the main concerns for the design of the controller. Moreover, ETC can reduce the communication among these agents without sacrificing their common goal. In [28], [94]–[96], the event-triggered consensus is achieved for agents using the sliding mode technique. This problem was also studied under the fault-tolerant scheme in [97].
A time-varying consensus algorithm using event-triggered SMC was developed in [98]. When the agents are governed by nonlinear dynamics, the problem of achieving consensus here is given in [99]. A distributed consensus algorithm for second-order agents was presented in [100], [101] whereas for underactuated systems under Markovian switching topology the distributed consensus problem is found in [100]. In [102], the authors presented the formation of spacecraft using only vision-based information with event-triggered communication. For heterogeneous agents, the consensus problem was solved in [103], for agents having nonholonomic constraints it is given in [104], [105], for multi-robot systems in [106], [107]. A discrete consensus algorithm was also studied under event-triggered communication between agents in [108].

C. Power Systems and Power Converters

As it is very well known that SMC is a widely accepted controller for power electronic converters due to its switching structure (see e.g. [109], [110] and the references therein). These control algorithms may not be very effective in terms of the resource utilization such as on-board power consumption, over computation, etc. An event-triggered SMC may be the right choice for these applications. In [29], [111], event-triggered SMC was designed for power converters, and in [112] the event-triggered algorithm was developed for load frequency control in an interconnected power system. The authors of [113] investigated the speed control of permanent magnet synchronous motor using event-triggered SMC. In a microgrid network, the event based SMC was designed to control the maximum power point tracking of photovoltaic generators in [114]. A leader-follower algorithm based islanding microgrid was developed in [115]. A reduced-order extended observer combined with event-triggered SMC was discussed in [116] for DC-DC converters. Different control strategies were developed for control of power flow in networks, such as adaptive event-triggered SMC in [117], [118], adaptive event-triggered integral high-order SMC in [119], distributed voltage control algorithm in [120], control of wind energy system [121].

D. Other Applications

There are many other applications of event-triggered SMC which have drawn attention towards it. The tracking problem of the mechanical system has been an interesting topic for industrial applications. The readers can refer to the references [59], [122]–[128]. A lot of research has been carried out for Markovian jump system which can be found in [123], [129], [130], [130]–[138], fault-tolerant control and estimation in [139]–[143], cyber-physical system in [144]–[151], control of space crafts in [152]–[156], fuzzy systems or neural network-based systems in [157]–[162], switched systems in [83], [89], [160], [163], [164]. The event-triggered algorithm with SMC for systems with either state and/or actuator constraint is reported in [91], [92], [133], [165]. There are many real-world applications for which the controller was designed using SMC with event-triggering strategy, e.g., for 2-D Roesser systems in [166], [167], non-affine systems in [168], passive systems in [169], memristive systems in [170], [171], odor localization in [172], congestion control in [173], control of process plant in [174], control of a quadrotor [175]. Finally, the output feedback based control of a plant was studied in [176], design of event/self-triggered control with the nonsingular terminal surface was discussed in [177], [178].

VI. CONCLUDING REMARKS AND FUTURE DIRECTIONS

In this paper, the state of art development on the event-triggered SMC is presented to date. Both theoretical backgrounds and their applications to different areas are also included here. It is our firm belief that a thorough review of this topic would further propel a new direction of research in this area in the years to come. The presentation highlights the benefits of ETC to motivate the readers, and then it is followed by a series of sections on how to design SMC via event-triggered feedback. The emphasis is given to make the content self-sufficient so that the idea of the event-triggered SMC can be understood with more clarity by the readers easily. We are very hopeful that this survey paper will again serve as a reference guide for researchers.

Although the paper covers (almost) all recent developments in the event-triggered SMC, there are many scopes to investigate further, including practical applications that may again involve new challenges. Here, we briefly give an overview of the future directions that may be worthy of looking into.

1) Event Based High-Order SMC: High-order sliding mode provides a family of control algorithms ensuring finite-time convergence of the sliding variable and its successive derivatives [7]–[9]. It is a popular research area within the sliding mode community because of its high accuracy, possible chattering alleviation, smooth control signal, etc. [12]. However, there are only a few works on the event based implementation of (second) high-order SMC reported in the literature [26]. It can be a promising future research direction to further develop this topic. One problem that may be very interesting in the event-triggered context is the accuracy of the sliding mode when an event based implementation is realized.

2) State Estimation via Event Based Sliding Mode: The estimation of the plant states possibly in the presence of uncertainties is a long-standing problem in the sliding mode theory. Different forms of sliding mode observers have been developed for the robust state estimation [6], [179]. Also, there is a significant interest in robust differentiators to obtain the real time differentiation of a signal [12]. In practice, the digital implementation of observers or differentiators may affect the estimated or differentiated signals. In [180, Sec. 5], the authors considered an implementation of the discrete hybrid differentiator via an event based measurement sensor and it is shown that under some assumptions it is still possible to achieve the same accuracy as in the time-triggered based measurements. Also this topic represents a promising future research direction to further investigate.

3) Applications of Event-Triggered SMC: SMC is known for its robust performance in many practical applications including robotics, power converters, spacecraft, etc. This motivates the practitioners to design the event based SMC for achieving the desired control goals while minimizing resources...
utilization. Additionally, in modern applications such as cyber-physical systems and networked control systems it is crucial to further develop this class of control algorithms.

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