Distributed Passivity-Based Control of DC Microgrids

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***Abstract*— In this paper, we propose a new distributed passivity-based control strategy for Direct Current (DC) microgrids. The considered DC microgrid includes Distributed Generation Units (DGUs) sharing power through resistive-inductive distribution lines. Each DGU is composed of a generic DC energy source that supplies an unknown load through a DC-DC buck converter. The proposed control scheme exploits a communication network, the topology of which can differ from the topology of the *physical* electrical network, in order to achieve proportional (fair) current sharing using a consensus-like algorithm. Moreover, the proposed distributed control scheme regulates the average value of the network voltages towards the corresponding desired reference, independently of the initial condition of the controlled microgrid. Convergence to a desired steady state is proven and satisfactorily assessed in simulations.**

I. Introduction

The Distributed Generation (DG) and the possibility of storing energy are changing the paradigm of power generation, transmission and distribution [1]. Differently from the past, nowadays many agents of the electrical distribution network are indeed *prosumers*, playing an active role in the network by producing, as well as consuming, energy. Moreover, the DG has been proposed as a conceptual solution to $i)$ facilitate the integration of Renewable Energy Sources (RES) in order to reduce the carbon emissions, $ii)$ increase the energy efficiency by reducing the transmission power losses, $iii)$ improve the service quality by supplying high-priority loads when a portion of the distribution network is isolated from the main grid and $iv)$ contain the costs for electrifying remote areas or re-powering the existing power networks due to the ever increasing energy demand. A low-voltage electrical distribution network composed of multiple Distributed Generation Units (DGUs), loads and energy storage devices interconnected through power lines is identified in the literature as a *microgrid* [2].

In the last decades, since most of the existing power networks are Alternate Current (AC)-based, the literature on power networks mainly considered AC grids (see for instance [3]–[6] and the references therein). However, the recent widespread use of RES as DGUs is motivating the design and operation of Direct Current (DC) microgrids. Several devices (e.g. photovoltaic panels, batteries, electronic appliances, electric vehicles) can indeed be directly connected to a DC network avoiding lossy DC-AC conversion stages and the issues related to the frequency and reactive power control [7]. Besides the development of industrial, commercial and residential DC distribution networks, some examples of existing or promising DC microgrid applications are ships, mobile military bases, trains, aircrafts and charging facilities for electric vehicles. For all these reasons, control of DC microgrids is recently gaining growing interest.

One of the main control objectives in DC microgrids is the regulation of the network voltage towards the nominal value that guarantees a proper functioning of the connected loads [8]–[10]. Additionally, in order to perform an efficient demand and supply matching and avoid the overstressing of a source, it is generally desired that the total demand of the microgrid is shared among all the DGUs proportionally to the generation capacity of their corresponding energy sources [11]. However, achieving current or power sharing prescribes the value of the required differences in voltages among the nodes of the network. As a consequence, it is generally not possible to control the voltage at each node towards the corresponding desired value. Then, in [11], [12] the authors propose to control the average voltage across the whole microgrid (not a specific node) towards a global voltage set point (e.g., the average of the voltage references).

In the literature, the aforementioned objectives are conventionally achieved by designing hierarchical and distributed control schemes requiring that each node of the *physical* network shares information through a communication network (*cyber* system). For the sake of feasibility, it is usually desired that $i)$ the control scheme is independent of the knowledge of the whole microgrid and $ii)$ each node of the microgrid communicates only with its neighbouring nodes. This motivated a growing interest in the development of distributed controllers, particularly aiming at current (load) sharing [11]–[16].

A. Main contributions

In this paper, we design a distributed passivity-based control (PBC) scheme that provably guarantees to achieve at the steady state proportional (fair) current sharing and average voltage regulation for DC microgrids that include buck converters, unknown "ZIP" (constant impedance, constant current, constant power) loads and dynamic resistive-inductive lines. Our main contributions are outlined below, where we also provide a brief comparison with existing theoretical results considering both the aforementioned control objectives:

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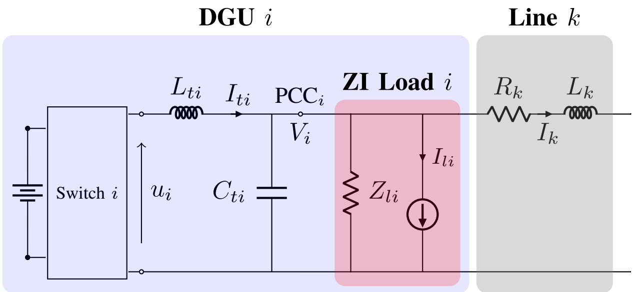


Fig. 1. Electrical scheme of DGU i and line k.

1) The considered microgrid model takes into account a possible meshed microgrid topology, incorporating dynamic resistive-inductive lines, which are neglected in e.g. [12] and [13], where purely resistive lines are adopted. Moreover, we include ZIP loads, which are not considered in e.g. [12], [15] and [16], where only constant current loads are analyzed.

2) The proposed control scheme requires only local measurements of the generated current. Differently from [12], [13] and [16], average voltage regulation is achieved without voltage measurements, while current sharing is achieved by exploiting a communication network where each DGU shares the corresponding value of the generated current with its neighbours. Notably, the design of the communication network is independent from the topology of the microgrid, in contrast to the results provided in [12], where an assumption is introduced on the product between the Laplacian matrices associated to the microgrid and communication networks [12, Assumption 4].

3) The stability analysis provides conditions on the controller gains making the control synthesis simpler than e.g. the one proposed in [12], where a Linear Matrix Inequality (LMI) problem is solved for each local primary voltage controller.

4) Convergence to a desired steady state is guaranteed, independently from the initial condition of the states of the physical microgrid and the controller. This is in contrast to e.g. [13], where a suitable initialization of the voltages is assumed, or [12] and [16] where a suitable initialization of the controller state is required to perform average voltage regulation.

B. Outline

The remainder of this paper is organized as follows. The microgrid model is presented in Section II, while the control problem is formulated in Section III. In Section IV, the proposed PBC control scheme is introduced and the stability of the controlled microgrid is studied. In Section V, the simulation results are illustrated and discussed, and finally, conclusions are gathered in Section VI.

II. DC Microgrid Model

In this paper, we study a typical DC microgrid composed of n Distributed Generation Units (DGUs) connected to each other through m resistive-inductive (RL) power lines. A schematic electrical diagram of the considered DC network including a DGU and a distribution line is illustrated in Fig. 1 (see also Table I for the description of the used symbols). Each DGU includes a DC-DC buck converter equipped with an output low-pass filter L_tC_t supplying an unknown “ZI” (constant impedance, constant current) load*. The DC load is connected to the so-called Point of Common Coupling (PCC). By using the Kirchhoff’s current (KCL) and voltage (KVL) laws, the equations describing the dynamic behaviour of the DGU i are given by

TABLE I

Description of the used symbols

	State variables
I_{ti}	Generated current
V_i	Load voltage
I_k	Line current
	Input
u_i	Control input (converter output voltage)
	Parameters
L_{ti}	Filter inductance
C_{ti}	Filter capacitor
R_k	Line resistance
L_k	Line inductance
	ZI Load
Z_{li}	Constant impedance
I_{li}	Constant current

$$
\begin{aligned}
L_{ti}\dot{I}_{ti} &= -V_i + u_i \\
C_{ti}\dot{V}_i &= I_{ti} - \frac{V_i}{Z_{li}} - I_{li} - \sum_{k\in\mathcal{E}_i} I_k,
\end{aligned}
\tag{1}
$$

where $\mathcal{E}_i$ is the set of distribution lines incident to the DGU i, while the control input u_i represents the buck converter output voltage†. The current shared among DGU i and DGU j is denoted by I_k, and its dynamic is given by

$$
L_k\dot{I}_k = (V_i - V_j) - R_k I_k. \tag{2}
$$

The symbols used in (1) and (2) are described in Table I.

The overall DC microgrid is represented by a connected and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the nodes, $\mathcal{V} = \{1, ..., n\}$, represent the DGUs and the edges, $\mathcal{E} = \{1, ..., m\}$, represent the distribution lines interconnecting the DGUs. The microgrid topology is described by its corresponding incidence matrix $\mathcal{B} \in \mathbb{R}^{n\times m}$. The ends of edge k are arbitrarily labeled with a $+$ and a $-$, and the

*Because of the page limitation, we restrict the analysis to ZI loads. However, the inclusion of constant power loads is briefly discussed in Remark 6.

†Note that, without loss of generality, in (1) we use u_i instead of $\delta_i V_{DCi}$, where δ_i is the duty cycle of the converter i and V_{DCi} is the constant DC voltage provided by a generic (voltage) energy source at node i.

entries of $\mathcal{B}$ are given by

$$\mathcal{B}_{ik} = \begin{cases} +1 & \text{if } i \text{ is the positive end of } k \\ -1 & \text{if } i \text{ is the negative end of } k \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Consequently, the overall dynamical system describing the microgrid behaviour can be written compactly for all DGUs $i \in \mathcal{V}$ as

$$\begin{aligned} L_t \dot{I}_t &= -V + u \\ L\dot{I} &= -RI - \mathcal{B}^\top V \\ C_t \dot{V} &= I_t + \mathcal{B}I - Z_l^{-1}V - I_l, \end{aligned} \tag{4}$$

where $I_t, I_l, V, u \in \mathbb{R}^n$, and $I \in \mathbb{R}^m$. Moreover, $C_t, L_t, Z_l \in \mathbb{R}^{n\times n}$ and $R, L \in \mathbb{R}^{m\times m}$ are positive definite diagonal matrices, e.g., $C_t = \operatorname{diag}(C_{t1}, \ldots, C_{tn})$.

Remark 1: (**Kron reduction**). Note that in (1), the loads are located at the Point of Common Coupling (PCC) of each DGU (see also Figure 1). This configuration can generally be obtained by a Kron reduction of the original network (with arbitrary interconnections of generation and load nodes), yielding an equivalent representation of the network [17].

III. Problem Formulation: Current Sharing and Voltage Regulation

In this section, we formulate two common control objectives in DC microgrids. First, we notice that for given constant input $\overline{u}$, a steady state solution $(\overline{I}_t, \overline{I}, \overline{V})$ to system (4) satisfies

$$\overline{V} = \overline{u} \tag{5a}$$

$$\overline{I} = -R^{-1}\mathcal{B}^\top \overline{V}. \tag{5b}$$

$$\overline{I}_t = -\mathcal{B}\overline{I} + Z_l^{-1}\overline{V} + I_l \tag{5c}$$

Equation (5c) implies[‡] that at the steady state the total generated current $\mathbb{1}^\top \overline{I}_t$ is equal to the total current $\mathbb{1}^\top(Z_l^{-1}\overline{V} + I_l)$ demanded by ZI loads. In order to achieve an efficient demand and supply matching, so avoiding the overstressing of a source, it is generally desired that the total demand of the microgrid is shared among all the DGUs proportionally to the generation capacity of their corresponding energy sources (fair current sharing). This desire is equivalent to achieve $w_i \overline{I}_{ti} = w_j \overline{I}_{tj}$ for all $i, j \in \mathcal{V}$, where a relatively large value of w_i corresponds to a relatively small generation capacity of DGU i. Consequently, we formulate the first control objective concerning with the steady state value of the generated currents $\overline{I}_t$:

Objective 1: (**Current sharing**).

$$\lim_{t\to\infty} I_t(t) = \overline{I}_t = W^{-1}\mathbb{1}i_t^*, \tag{6}$$

with $W = \operatorname{diag}(w_1, \ldots, w_n)$, $w_i > 0$, for all $i \in \mathcal{V}$ and i_t^* any scalar.

Note that the steady state requirement $\mathbb{1}^\top \overline{I}_t = \mathbb{1}^\top(Z_l^{-1}\overline{V} + I_l)$ implies that $i_t^* = \mathbb{1}^\top(Z_l^{-1}\overline{V} + I_l)/(\mathbb{1}^\top W^{-1}\mathbb{1})$. Before formulating the second control objective concerning with the steady state value of the PCC voltages $\overline{V}$, we assume that for every DGU i, there exists a nominal reference voltage $V_i^\star$:

Assumption 1: (**Nominal voltages**). There exists a reference voltage[§] $V_i^\star \in \mathbb{R}_{>0}$ at the PCC, for all $i \in \mathcal{V}$.

Achieving Objective 1 prescribes the value of the required differences in voltages among the nodes of the network. As a consequence, it is generally not possible to control the voltage at each node towards the corresponding desired value. Followig [11], [12], we aim at achieving (weighted) *average voltage regulation*, where the sources with the largest generation capacity determine the grid voltage. Then, we select a weight of $1/w_i$ for all $i \in \mathcal{V}$, leading to the second objective:

Objective 2: (**Average voltage regulation**).

$$\lim_{t\to\infty} \mathbb{1}^\top W^{-1}V(t) = \mathbb{1}^\top W^{-1}\overline{V} = \mathbb{1}^\top W^{-1}V^\star. \tag{7}$$

IV. The Proposed Solution: A Passivity-Based Approach

In this section, we introduce the key aspects of the proposed solution to simultaneously achieve Objective 1 and Objective 2, consisting of a passivity-based distributed control algorithm. To permit the controller design, the following assumption is introduced on the available information of system (4):

Assumption 2: (**Available informations**). The current I_{ti} is measurable at DGU $i \in \mathcal{V}$.

Before proposing a distributed controller achieving the objectives discussed in the previous section, we study the passivity property of system (4), proposing a Krasovskii-type storage function (see [18]–[21] for more details), which depends on the first time derivate of the states of system (4). For this reason, we consider the following *extended dynamics*[¶] of system (4)

$$\begin{aligned} L_t \dot{I}_t &= -V + u \\ L\dot{I} &= -RI - \mathcal{B}^\top V \\ C_t \dot{V} &= I_t + \mathcal{B}I - Z_l^{-1}V - I_l \\ L_t \ddot{I}_t &= -\dot{V} + \upsilon \\ L\ddot{I} &= -R\dot{I} - \mathcal{B}^\top \dot{V} \\ C_t \ddot{V} &= \dot{I}_t + \mathcal{B}\dot{I} - Z_l^{-1}\dot{V} \\ \dot{u} &= \upsilon. \end{aligned} \tag{8}$$

Then, the following result can be proved.

Lemma 1: (**Passivity property of** (8)). Let Assumptions 1 and 2 hold. System (8) is passive with respect to the supply rate $\upsilon^\top \dot{I}_t$ and the storage function

$$S_1(\dot{I}_t, \dot{I}, \dot{V}) = \frac{1}{2}\dot{I}_t^\top L_t \dot{I}_t + \frac{1}{2}\dot{I}^\top L\dot{I} + \frac{1}{2}\dot{V}^\top C_t \dot{V}. \tag{9}$$

‡The incidence matrix $\mathcal{B}$, satisfies $\mathbb{1}^\top \mathcal{B} = \mathbf{0}$, where $\mathbb{1} \in \mathbb{R}^n$ is the vector consisting of all ones.

§Often the values for $V_i^\star$ are chosen identical for all $i \in \mathcal{V}$. However, the control scheme that we propose in Section IV permits to select also non-identical values for $V_i^\star$.

¶The state variables and the input of the extended system are $I_t, I, V, \dot{I}_t, \dot{I}, \dot{V}, u$ and υ, respectively.

Proof: A straightforward calculation shows that the storage function S_1 in (9) satisfies

$$\dot{S}_1 = -\dot{I}^\top R \dot{I} - \dot{V}^\top Z_l^{-1} \dot{V} + v^\top \dot{I}_t \leq v^\top \dot{I}_t, \qquad (10)$$

along the solutions to (8), which concludes the proof. ■

To permit the design of a distributed controller achieving Objective 1, we exploit a communication network where each DGU shares the information with its neighbouring DGUs. We make the following assumption on the communication network:

Assumption 3: (**Communication network**). The graph $\mathcal{G}^c = (\mathcal{V}, \mathcal{E}^c)$ corresponding to the topology of the communication network is undirected and connected, where $\mathcal{E}^c = \{1, ..., m_c\}$ represents the set of the communication links between the DGUs[‖].

Then, the communication network topology is described by its corresponding incidence matrix $\mathcal{B}^c \in \mathbb{R}^{n \times m_c}$, which is defined similarly to $\mathcal{B}$ in (3). Let $\mathcal{L}_c = \mathcal{B}^c \Gamma (\mathcal{B}^c)^\top$ be the (weighted) Laplacian matrix associated to the communication network, where $\Gamma \in \mathbb{R}^{m_c \times m_c}$ is a positive definite diagonal matrix describing the weights on the edges.

Remark 2: (**Suggested consensus protocol**). Since the output port-variable $\dot{I}_t$ is integrable, then Lemma 1 suggests as a candidate storage function $\mathcal{S} = S_1 + \sigma(I_t)$, where S_1 is given by (9) and the function $\sigma : \mathbb{R}^n \to \mathbb{R}_{>0}$ is chosen such that $\mathcal{S}$ has a minimum satisfying Objective 1. Then, $\sigma(I_t) = \frac{1}{2}(\mathcal{L}^c W I_t)^\top (\mathcal{L}^c W I_t)$ has a minimum at $\mathcal{L}^c W I_t = 0$, which implies $W I_t \in \operatorname{im}(\mathbb{1})$ (see Objective 1).

Consequently, Remark 2 suggests to augment system (4) with additional state variables (distributed integrators) θ_i, $i \in \mathcal{V}$, with dynamics given by

$$\dot{\theta}_i = \sum_{j \in \mathcal{N}_i^c} \gamma_{ij} (w_i I_{ti} - w_j I_{tj}), \qquad (11)$$

where $\mathcal{N}_i^c$ is the set of the DGUs that communicate with the DGU i, and $\gamma_{ij} = \gamma_{ji} \in \mathbb{R}_{>0}$ are the entries of Γ, i.e., the edge weights. Then, the dynamics in (11) can be expressed compactly for all nodes $i \in \mathcal{V}$ as

$$\dot{\theta} = \mathcal{L}^c W I_t, \qquad (12)$$

that indeed has the form of a consensus protocol, permitting a steady state where $W \overline{I}_t \in \operatorname{im}(\mathbb{1})$ (see Objective 1 and Remark 2).

Following the procedure suggested in [15], we now interconnect the dynamical *physical* system (4) with the dynamical *cyber* system (12) by choosing

$$u = -W \mathcal{L}^c \theta + u_c, \qquad (13)$$

yielding the following dynamical *cyber-physical* system

$$\begin{aligned} L_t \dot{I}_t &= -V - W \mathcal{L}^c \theta + u_c \\ L \dot{I} &= -RI - \mathcal{B}^\top V \\ C_t \dot{V} &= I_t + \mathcal{B} I - Z_l^{-1} V - I_l \\ \dot{\theta} &= \mathcal{L}^c W I_t, \end{aligned} \qquad (14)$$

[‖]Note that the topology of the communication network can differ from the topology of the physical network.

where u_c is the control input to be designed. Before designing u_c, we study the passivity property of system (14), proposing again a Krasovskii-type storage function. Consider first the extended dynamics[**] of system (14)

$$\begin{aligned} L_t \dot{I}_t &= -V - W \mathcal{L}^c \theta + u_c && (15a) \\ L \dot{I} &= -RI - \mathcal{B}^\top V && (15b) \\ C_t \dot{V} &= I_t + \mathcal{B} I - Z_l^{-1} V - I_l && (15c) \\ \dot{\theta} &= \mathcal{L}^c W I_t && (15d) \\ L_t \ddot{I}_t &= -\dot{V} - W \mathcal{L}^c \dot{\theta} + v_c && (15e) \\ L \ddot{I} &= -R \dot{I} - \mathcal{B}^\top \dot{V} && (15f) \\ C_t \ddot{V} &= \dot{I}_t + \mathcal{B} \dot{I} - Z_l^{-1} \dot{V} && (15g) \\ \ddot{\theta} &= \mathcal{L}^c W \dot{I}_t && (15h) \\ \dot{u}_c &= v_c. && (15i) \end{aligned}$$

Then, the following result can be proved.

Lemma 2: (**Passivity property of** (15)). Let Assumptions 1-3 hold. System (15) is passive with respect to the supply rate $v_c^\top \dot{I}_t$ and the storage function[††]

$$S_2(\dot{I}_t, \dot{I}, \dot{V}, \dot{\theta}) = S_1 + \frac{1}{2} \dot{\theta}^\top \dot{\theta}, \qquad (16)$$

where S_1 is given by (9).

Proof: A straightforward calculation shows that the storage function S_2 in (16) satisfies

$$\dot{S}_2 = -\dot{I}^\top R \dot{I} - \dot{V}^\top Z_l^{-1} \dot{V} + v_c^\top \dot{I}_t \leq v_c^\top \dot{I}_t, \qquad (17)$$

along the solutions to (15), which concludes the proof. ■

Remark 3: (**Average voltage regulation**). Note that, at steady state, after pre-multiplying both sides of (15a) by $\mathbb{1}^T W^{-1}$, one obtains $\mathbb{1}^T W^{-1} \overline{V} = \mathbb{1}^T W^{-1} \overline{u}_c$. As a consequence, regulating u_c towards $V^\star$ ensures that Objective 2 is achieved. We take this into account in the choice of the desired storage function in Theorem 1.

Before introducing the main result of this work, we assume that a steady state solution to system (15) exists:

Lemma 3: (**Existence of a unique steady state solution**). Let Assumptions 1 and 3 hold. Moreover, let $u_c = V^\star$ and $v_c = 0$. There exists a unique steady state solution $(\overline{I}_t, \overline{I}, \overline{V}, \overline{\theta}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}, V^\star)$ to system (15), satisfying Objective 1 and Objective 2.[‡‡]

We can now show that the solutions to (15) converge to a steady state, achieving Objective 1 and Objective 2.

Theorem 1: (**Main result**). Let Assumptions 1–3 hold. Consider system (15) controlled by

$$v_c = -K_d^{-1} \dot{I}_t - K_d^{-1} K_p (u_c - V^\star), \qquad (18)$$

[**]The state variables and the input of the extended system are $I_t, I, V, \theta, \dot{I}_t, \dot{I}, \dot{V}, \dot{\theta}, u_c$ and v_c, respectively.

[††]The storage function S_2 in (16) depends on the states $\dot{I}_t, \dot{V}, \dot{I}$ and $\dot{\theta}$. Consequently, according to [22, Remark 2], S_2 depends also on I_t, V, I, θ and u_c, i.e., the entire state of the auxiliary system (15).

[‡‡]Let $G := \mathcal{B} R^{-1} \mathcal{B}^\top + Z_l^{-1}$. It can be proved that $\overline{I}_t, \overline{I}, \overline{V}, \overline{\theta}$ satisfy $\overline{I}_t = G \overline{V} + I_l$, (5b), $\overline{V} = -W \mathcal{L}^c \overline{\theta} + V^\star$, $\overline{\theta} = \begin{bmatrix} L \\ \mathbb{1}^\top \end{bmatrix}^\dagger \begin{bmatrix} b \\ \mathbb{1}^\top \theta(0) \end{bmatrix}$, where $b := \mathcal{L}^c W (G V^\star + I_l)$ and $L := \mathcal{L}^c W G W \mathcal{L}^c$.

where $K_p, K_d \in \mathbb{R}^{n\times n}$ are positive definite diagonal matrices. Then, the solutions to system (15) controlled by (18) converge to the desired steady-state, satisfying Objective 1 and Objective 2.

Proof: Consider the desired storage function (see also Remark 3)

$$\mathcal{S} = S_2 + \frac{1}{2}(u_c - V^\star)^\top K_p (u_c - V^\star), \tag{19}$$

where S_2 is given by (16). It is immediate to see that $\mathcal{S}$ attains a minimum at $(\overline{I}_t, \overline{I}, \overline{V}, \overline{\theta}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}, V^\star)$. Furthermore, $\mathcal{S}$ satisfies

$$\begin{aligned}\dot{\mathcal{S}} &= -\dot{I}^\top R \dot{I} - \dot{V}^\top Z_l^{-1} \dot{V} + \dot{u}_c^\top \dot{I}_t + (u_c - V^\star)^\top K_p \dot{u}_c \\ &= -\dot{I}^\top R \dot{I} - \dot{V}^\top Z_l^{-1} \dot{V} + \dot{u}_c^\top (\dot{I}_t + K_p(u_c - V^\star)) \\ &= -\dot{I}^\top R \dot{I} - \dot{V}^\top Z_l^{-1} \dot{V} - \dot{u}_c^\top K_d \dot{u}_c \end{aligned} \tag{20}$$

along the solutions to (15). According to LaSalle's invariance principle, the solutions to (15) approach the largest invariant set contained entirely in the set

$$\Upsilon = \left\{I_t, I, V, \theta, \dot{I}_t, \dot{I}, \dot{V}, \dot{\theta}, u_c : \dot{I} = \mathbf{0}, \dot{V} = \mathbf{0}, \dot{u}_c = \mathbf{0}\right\}, \tag{21}$$

implying that, on this set Υ, $I = \overline{I}, V = \overline{V}$ and $u_c = \overline{u}_c$ are constant vectors. Furthermore, on this set Υ, it follows from (15g) that $\dot{I}_t = \mathbf{0}$, i.e., also $I_t = \overline{I}_t$ is a constant vector. Then, from (18) it follows that, on this set Υ, $\overline{u}_c = V^\star$. Furthermore, on this set Υ, equation (15e) satisfies $W\mathcal{L}^c\dot{\theta} = \mathbf{0}$, i.e., $\dot{\theta} = \mathbb{1}\alpha, \alpha \in \mathbb{R}$ (or equivalently $\mathcal{L}^c W \overline{I}_t = \mathbb{1}\alpha$). Consequently, $\mathbb{1}^\top \mathcal{L}^c W \overline{I}_t = \mathbb{1}^\top \mathbb{1}\alpha$ implies $\alpha = 0$. This further implies $\mathcal{L}^c W \overline{I}_t = \mathbf{0}$, achieving Objective 1, and $\dot{\theta} = \mathbf{0}$, i.e., $\theta = \overline{\theta}$ is a constant vector. Finally, pre-multiplying both sides of (15a) by $\mathbb{1}^T W^{-1}$, one obtains $\mathbb{1}^T W^{-1} \overline{V} = \mathbb{1}^T W^{-1} V^\star$, achieving Objective 2. ■

Remark 4: (**Alternative controller**). Note that controller (18) requires the information of $\dot{I}_t$, which could be affected by error measurements. In order to avoid this, controller (18) can be replaced by

$$\begin{aligned} u_c &= \phi - K_d^{-1} I_t \\ K_d \dot{\phi} &= -K_p (u_c - V^\star). \end{aligned} \tag{22}$$

Then, the same results of Theorem 1 can be straightforwardly proved.

Remark 5: (**Robustness and Plug-and-Play**). Note that the proposed controller (18) (or (22)) does not require the information of the load, and the stability analysis in Theorem 1 does not depend on the value of Z_l and I_l. Moreover, even if we assume a constant network topology, since the convergence result of Theorem 1 holds globally, independently of the initial conditions of the physical power network and the controller state, the proposed solution is expected to show Plug-and-Play capabilities. However, the analysis of the corresponding switched system is outside the scope of this work.

Remark 6: (**ZIP loads**). Similarly to [22, Lemma 1], if constant power loads are also considered, it can be proved that Lemma 1, Lemma 2 and Theorem 1 hold *locally* in a neighborhood of the equilibrium, where the trajectories of the controlled system satisfy $Z_{li}^{-1} - V_i^{-2} P_{li} > 0$ for all $i \in \mathcal{V}$, P_{li} being the constant power demand of load i.

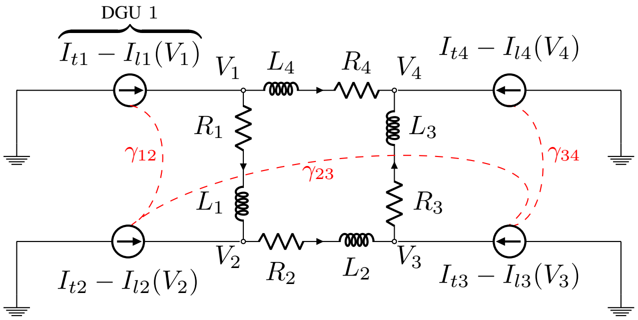


Fig. 2. Scheme of the considered (Kron reduced) microgrid with 4 power converters. The dashed lines represent the communication network and $I_{li}(V_i) = Z_{li}^{-1} V_i + I_{li}$.

TABLE II
MICROGRID PARAMETERS

DGU		1	2	3	4
L_{ti}	(mH)	1.8	2.0	3.0	2.2
C_{ti}	(mF)	2.2	1.9	2.5	1.7
w_i	–	0.4^{-1}	0.2^{-1}	0.15^{-1}	0.25^{-1}
$V_i^\star$	(V)	380.0	380.0	380.0	380.0
$Z_l(0)$	(Ω)	16.7	50.0	16.7	20.0
$I_l(0)$	(A)	30.0	15.0	30.0	26.0
ΔI_l	(A)	10.0	7.0	−10.0	5.0

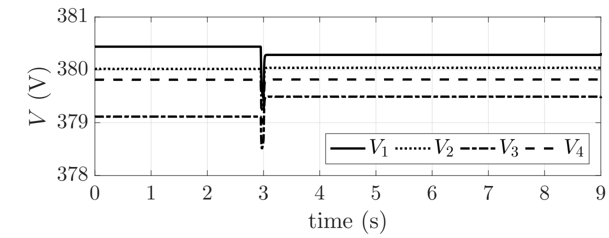
TABLE III
LINE PARAMETERS

Line		1	2	3	4
R_k	(mΩ)	70	50	80	60
L_k	(μH)	2.1	2.3	2.0	1.8

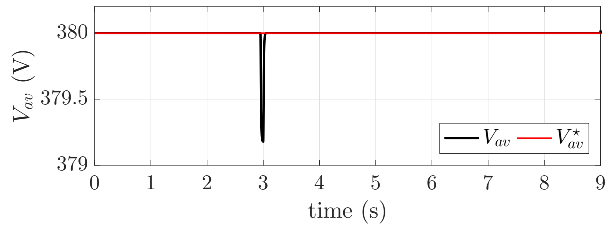
V. SIMULATION RESULTS

In this section, the control strategy proposed in Section IV is assessed in simulation. We consider a microgrid composed of four DGUs interconnected as shown in Figure 2, where also the communication network is represented. The parameters of each DGU and the line parameters are reported in Tables II and III, respectively. The weights associated with the edges of the communication graph are $\gamma_{12} = \gamma_{23} = \gamma_{34} = 1 \times 10^2$. In the controller (18), we have selected $K_d = \mathbb{I}_4$ and $K_p = 100 \times \mathbb{I}_4$, $\mathbb{I}_4 \in \mathbb{R}^{4\times 4}$ being the identity matrix.

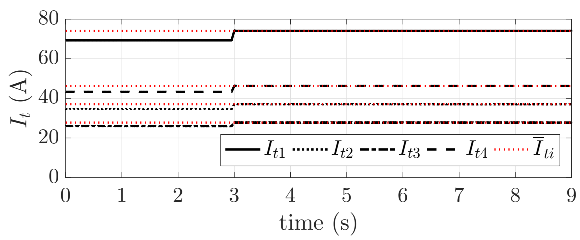
The system is initially at a steady state with load impedance $Z_l(0)$ and current $I_l(0)$. Then, consider a load current variation ΔI_l at the time instant $t = 3\,\mathrm{s}$ (see Table II). The PCC voltages and the average voltage of the network are illustrated in Figure 3 (a) and (b), respectively. One can appreciate that the steady state weighted average of the PCC voltages (denoted by V_{av}) is equal to the weighted average of the corresponding references (see Objective 2). Figure 3 (c) shows that the current generated by each DGU converges to the desired value, achieving proportional current sharing (see Objective 1), while Figure 3 (d) illustrates the currents shared among the DGUs through the lines of the network.



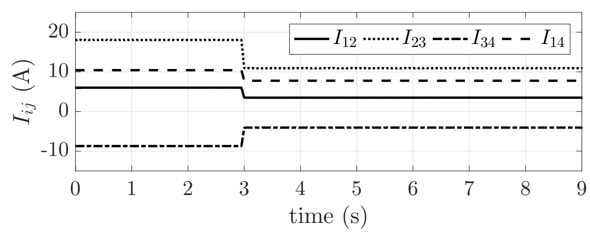
(a) Voltages



(b) Weighted average voltage



(c) Generated currents



(d) Line currents

Fig. 3. Time evolution of the voltage at the PCC of each DGU (a); weighted average value of the microgrid voltages together with the corresponding reference (b); current generated by each DGU together with the corresponding values (dashed lines) corresponding to (proportional) current sharing for $t > 3$ (c); currents shared among DGUs (d).

VI. Conclusions

In this paper, a distributed passivity-based control scheme is proposed for achieving at the steady state proportional (fair) current sharing and regulating the average value of the voltages of a DC microgrid that includes buck converters, unknown "ZIP" (constant impedance, constant current, constant power) loads and dynamic resistive-inductive lines. The control objectives are achieved by designing a consensus-like protocol requiring that each node of the microgrid (*physical* system) shares information with its neighbouring nodes through a communication network (*cyber* system). The controlled *cyber-physical* system is proven to converge globally to a desired steady state, independently of the initial conditions of the system states. Interesting future research includes the analysis of different converter types, such as boost converters [8], [10].

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