

Advanced and Optimization Based Sliding Mode Control

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Advanced and Optimization Based Sliding Mode Control Theory and Applications

A. Ferrara

University of Pavia
Pavia, Italy

G. P. Incremona

Politecnico di Milano
Milan, Italy

M. Cucuzzella

University of Groningen
Groningen, the Netherlands

siam[®]

Society for Industrial and Applied Mathematics
Philadelphia

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*To our wonderful families,
who have constantly supported and endured us
during the time we have dedicated to the writing of this book*

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Notation

We chose to use the symbols in the book according to the following conventions. Scalar values or signals are denoted by lowercase letters such as x , a , and t . Vectors are indicated by boldface and italic letters such as \mathbf{x} and \mathbf{y} and matrices by boldface and italic uppercase letters such as \mathbf{A} and \mathbf{B} . The elements x_1, \dots, x_n of a vector \mathbf{x} or $a_{11}, a_{12}, \dots, a_{mn}$ of a matrix \mathbf{A} are represented by italics. Let \otimes indicate the Kronecker product. The r th time derivative of a signal $x(t)$, with $r > 2$, is denoted by $x^{(r)}(t)$. Sets are symbolized by calligraphic letters such as \mathcal{F} and \mathcal{X} . The inequality $\mathbf{P} > \mathbf{0}$ is interpreted in two different ways: referring to optimal control or linear matrix inequalities, $\mathbf{P} > \mathbf{0}$ means that the matrix \mathbf{P} is positive definite; alternatively, we interpret the sign $>$ as an elementwise relation to indicate that all elements of the matrix \mathbf{P} are positive (i.e., $p_{ij} > 0$ for all i, j). Analogously, the symbol \sim indicates similarity between two matrices or the Pontryagin difference between two sets. For any symmetric matrix \mathbf{A} , $\lambda_{\max}(\mathbf{A})$ and $\lambda_{\min}(\mathbf{A})$ denote the largest and the smallest eigenvalues of matrix \mathbf{A} , respectively. Given a matrix $\mathbf{M} \in \mathbb{R}^{n \times m}$ with $n > m$, its orthogonal complement is $\mathbf{M}^\perp \in \mathbb{R}^{n \times (n-m)}$, while $\mathbf{M}^\dagger \in \mathbb{R}^{n \times m}$ is its Moore–Penrose pseudoinverse. Let $\mathbf{x} \cdot \mathbf{y}$ denote the vector dot product and $\mathbf{x} \times \mathbf{y}$ denote the cross product, while, given two functions $f(\cdot)$ and $g(\cdot)$, let $f(\cdot) \circ g(\cdot) = f(g(\cdot))$ denote the function composition. Let $x(k+i|k)$ denote the value of x at the time instant $k+i$ predicted at the time instant k . Given a generic signal w , let $w_{[t_1, t_2]}$ be a signal defined from time t_1 to time t_2 . Note that, to simplify the notation, when it is obvious from the context, we omit the subscript $[t_1, t_2]$. The symbol id represents the identity function from \mathbb{R} to \mathbb{R} . Finally, we use $\|\cdot\|$ to denote the Euclidean norm, $\|\cdot\|_\infty$ to denote the infinity norm, $|\cdot|$ to denote the absolute value, and $\|\cdot\|_W^2$ to denote the squared norm weighted by the matrix \mathbf{W} . The set of signals w , the values of which belong to a compact set \mathcal{W} , is denoted by $\mathcal{M}_{\mathcal{W}}$, while $\mathcal{W}^{\sup} := \sup_{w \in \mathcal{W}} \{\|w\|\}$.

Preface

This book deals with sliding mode control (SMC) of uncertain nonlinear systems, focusing in particular on advanced and optimization based algorithms. The aim is to provide an overview and critical discussion of the results published by the authors in recent years, so as to organize them into a well-structured and consistent compendium. The book includes a survey of classical SMC theory, along with the introduction of four different families of advanced original SMC methods. Specifically, the new methods discussed in the book are optimization based higher order sliding mode control (HOSM), integral HOSM control, constrained HOSM, and networked event-triggered sliding mode control (ET-SMC).

In the first part of the book, after a tutorial review of classical SMC theory, we introduce and theoretically analyze the advanced algorithms. We provide numerical results to complement the theoretical treatment. In the second part of the book, we discuss applications of the considered algorithms as case studies. The application problems involve complex robotic systems and microgrids. We report simulation and experimental results in the book.

While the classical SMC theory can also be found in other research books and textbooks for graduate and postgraduate courses of study, the advanced algorithms presented in this book were previously published only in journal papers. Moreover, they have never been organized systematically as in this book, which will make these methods much more understandable and usable by the scientific community. We discuss algorithms by adopting common notation and referring, when possible, to the same illustrative example. This will help the reader compare the advantages and limitations of the various approaches individually, but also identify, in any field implementation, the advanced SMC algorithm that is more suitable to use.

The real applications considered in this book belong to the class of complex systems because their models are nonlinear, they are affected by significant uncertainties, and their state and inputs must comply with constraints. Such application examples often operate in the presence of communication networks, which are intrinsic elements of the control loop. They are taken as examples of typical real-world complex plants. By virtue of the considered applications, this book can be a useful guide to practitioners, providing practical rules for developing field implementations of the algorithms. The application examples also constitute a significant proof of the validity of the advanced control concepts for the benefit of researchers.

This book is aimed at a general readership, including students, researchers, and practitioners with basic knowledge in control engineering, process physics, and applied mathematics. The authors also hope it will be useful and interesting in courses on advanced automatic control for undergraduate and graduate students.

A. Ferrara, G. P. Incremona, and M. Cucuzzella